International Institute for
Technology and Management
Tutoring Sheet # 2b
Unit 04b   :   Statistics 2

1.   i. From your tables find the probability that there are at least 6 successes for the binomial distribution with 17 trials and probability of success 0.35.

ii. From your tables find the probability that $X > 7$ when X has a Poisson distribution with mean 4.5.

2. Consider two random variables X and Y. X can take the values 0, 1 and 2 and Y can take the values 1 and 2. The joint probabilities for each pair are given by the following table.

	$X = 0$	$X = 1$	$X = 2$
$Y = 1$	0.1	0.2	0.3
$Y = 2$	0.1	0.1	0.2

Let $Z = \max(X, Y)$ be the larger of the two variables. Find $E(Y)$, $E(Y|X = 1)$, $E(Z)$ and $E(Z|X = 1)$.

3. The random variable X has a density function given by

$$f(x) = \frac{3x^2 + 2x}{2}$$

defined over the region $0 \le x \le 1$. Find $\Pr(X > 0.8 | X > 0.6)$, $E(X)$, $\text{Var}(X)$ and $\text{Cov}\left(X, \frac{1}{X}\right)$.

4. **The random variable X is normally distributed with mean 0 and variance 9.**

Find $\Pr(X > 3.6 \mid X > 1.8)$ and $\Pr(|X| > 3.6 \mid |X| > 1.8)$

5. Consider two random variables X and Y. X can take the values -1, 0 and 1 and Y can take the values 0, 1 and 2. The joint probabilities for each pair are given by the following table.

	$X = -1$	$X = 0$	$X = 1$
$Y = 0$	0.1	0.2	0.1
$Y = 1$	0.1	0.05	0.1
$Y = 2$	0.1	0.05	0.2

a) Calculate the marginal distributions and expected values of X and Y.

(9 marks)

b) Calculate the covariance of the random variables U and V, where $U = X + Y$ and $V = X - Y$.

(7 marks)

c) Calculate $E(V|U = 1)$

(7 marks)

d) The random variable W has the same distribution as X and the random variable Z has the same distribution as Y. The random variables W and Z are independent. Write down the table for the joint probabilities of W and Z and calculate their covariance.

(7 marks)

6. The random variable X has density function given by $f(x) = \dfrac{12x^2(x+1)}{7}$

   defined over the region $0 < x < 1$.
   (a) Calculate $\Pr(X > 0.5 \mid X > 0.25)$ and $E(X)$.
   (b) Calculate $\text{Cov}(\dfrac{1}{1+X}, \dfrac{1}{X^2})$

7. Consider two random variables X and Y. They both take the values 0, 1 and 2. The joint probabilities for each pair are given by the following table.

	$X = 0$	$X = 1$	$X = 2$
$Y = 0$	0.10	0.06	0.14
$Y = 1$	0.08	0.06	0.16
$Y = 2$	0.20	0.08	0.12

   (a) Calculate the marginal distributions, and the expected values of X and Y.
   (b) Calculate $E(X \mid Y = 1)$ and $E(X \mid X + Y = 3)$.
   (c) Define $U = |X - 1|$ and $V = Y$. Calculate the covariance of U and V.
   (d) Are U and V are independent variables? Explain your answer.

8. Consider random variables X and Y with joint density function
$$f_{XY}(x, y) = \begin{cases} k(3x - 2) & 0 < y < x < 5 \\ 0 & otherwise \end{cases}$$

(a) Find k.

(b) Find $f_X(x)$. Hence evaluate $E(X)$.

(c) Write down an expression for $f_{Y|X}(y|x)$. Find $E(Y \mid X)$ and hence evaluate $E(Y)$.

(d) Evaluate $P(2Y > X)$.