

International Institute for  
Technology and Management



## Tutoring Sheet # 2a

Unit 04b : Statistics 2

1. The discrete random variable  $X$  has the probability distribution specified in the following table:

x	-1	0	1	2
$P(X = x)$	0.25	0.10	0.45	0.20

- a. Find  $P(-1 < X < 1)$  ,  $P(-1 \leq X < 1)$   
b. Find  $E(2X + 3)$
2. A bag contains 3 red balls and 1 blue ball. A second bag contains 1 red ball and 1 blue ball. A ball is picked out of each bag and is then placed in the other bag. What is the expected number of red balls in the first bag? (Assume that the balls are taken from each bag simultaneously).
3. For a discrete r.v.  $X$  , the c.d.f ,  $F(X)$  is as shown:

x	1	2	3	4	5
$F(x) = P(X \leq x)$	0.2	0.32	0.67	0.9	1

- a. Find  $P(X = 3)$                       b.  $P(X > 2)$
4. The continuous r.v.  $X$  has p.d.f. :
- $$f(x) = \begin{cases} k(x+2)^2 & -2 \leq x < 0 \\ 4k & 0 \leq x \leq 4/3 \\ 0 & \text{Otherwise} \end{cases}$$
- a.) Find the value of the constant  $k$  and sketch the graph of  $y=f(x)$   
b.) Find  $P(-1 \leq X \leq 1)$   
c.) Find  $P(X > 1)$
5. The continuous r.v.  $X$  has p.d.f. ,  $f(x) = 0.75(1+x^2)$  ,  $0 \leq x \leq 1$   
If  $E(X) = \mu$  and  $\text{Var}(X) = \sigma^2$  , find  $P(|X - \mu| < \sigma)$

6. If  $X$  is a continuous r.v. with p.d.f,  $f(x) = \frac{1}{8}x$ ,  $0 \leq x \leq 4$

- a.) Find the c.d.f.  $F(x)$
- b.) Find the median  $m$
- c.) Find  $P(0.3 \leq X \leq 1.8)$

7. The c.d.f.  $F(x)$  for a continuous r.v. is defined as:

$$F(x) = \begin{cases} 0 & x < 0 \\ 2x - 2x^2 & 0 \leq x \leq \frac{1}{4} \\ a + x & \frac{1}{4} \leq x \leq \frac{1}{2} \\ b + 2x^2 - x & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 1 & x \geq \frac{3}{4} \end{cases}$$

- a.) Find  $a$  and  $b$
- b.) Find and sketch the p.d.f.  $f(x)$
- c.) Find the mean  $\mu$

8. The random variable  $X$  has density function  $f(x)$  given by  $f(x) = 3e^{-3x}$  defined over the region  $x > 0$ , Let  $Y = e^X$ , find  $E(Y)$  and  $\text{Var}(Y)$ .

9. A random variable can only take the values 0 and 2, its variance is 1, find its mean.

10.  $X$  is a random variable with  $E(X^2) = 3.6$  and  $P(X = 2) = 0.6$  and  $P(X = 3) = 0.1$ . The random variable  $X$  takes just one other value besides 2 and 3.

- i. What is the other value that  $X$  takes?
- ii. What is the variance of  $X$ ?

11.  $X$  is a random variable with density function

$$f_X(x) = 3(1+x)^2/19$$

over the range  $(1, 2)$ . Using calculus, find the mean and the expected value of  $1/(1+X)$ . What is  $P(X > 1.5)$ ?

12.  $X$  is a random variable with density function

$$f_X(x) = a + bx$$

over the range  $(0, 1)$ , where  $a, b$  are constants. The mean of  $X$  is 0.5.

- i. Find  $a, b$ .
- ii. Find the cumulative distribution function of  $X$ .
- iii. Find the variance of  $X$ .