04b Sample Examination Problems Chapter 2

1. Alphonse, Bertille and Clarence play a simple game with a fair die. Alphonse tosses the die and observes the number on the uppermost face, which the other two do not see. Bertille tries to guess that number. If she is right, she wins. If she is wrong, then Clarence tries to guess the number and so win the game. If neither Bertille nor Clarence guess correctly, then Alphonse wins the game.

   (a) What is the chance that Clarence wins the game?

   (b) What is the conditional probability that Clarence wins the game given that Alphonse does not win?

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2. In a large sociology class, 10% of all students get an A grade in their end-of-year examination. 60% of all students had missed no classes. The examiners checked and found that 10% of all students who had got an A grade had missed no classes.

   (a) What is the probability that a student with an A grade had missed no classes?

   (b) What is the probability that a student who missed at least one class did not get an A grade?

   (c) Are the events 'got an A grade' and 'missed no classes' mutually exclusive? Explain.

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3. State and prove Bayes' Theorem.

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4. In six independent tosses of a fair coin, what is the probability that there are at least three successive heads somewhere in the sequence?

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5. If A and B are events such that $P(A|B^c) = 2P(A|B)$ and $P(B^c) = 2P(B)$, show that $P(B^c|A) = 0.8$. (The event B^c is the complement of event B.)

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6. Show that if $P(A \mid B) = P(A \mid B^c)$ then A and B are independent.

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7. A box of 55 coloured light bulbs consists of 5 blue bulbs, 20 pink bulbs, 20 yellow bulbs and 10 white bulbs. If 4 bulbs are selected at random without replacement, what is the probability that:

   (a) all 4 bulbs are white

   (b) all 4 bulbs are the same colour

   (c) all 4 bulbs are different colours

   (d) no bulb is yellow?

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8. Annabel is twice as likely to go shopping as Barbara. Carmel is three times as likely to go shopping when Annabel meets her than she is when Barbara meets her. How much more likely is Annabel to shop than Barbara when each has been met by Carmel?