Q7 2005 ZA

7. The rate of sales (S) of a newly launched product is expected to be given by the following function of time (T) from launch:

$$S = T(1+\cos T)$$

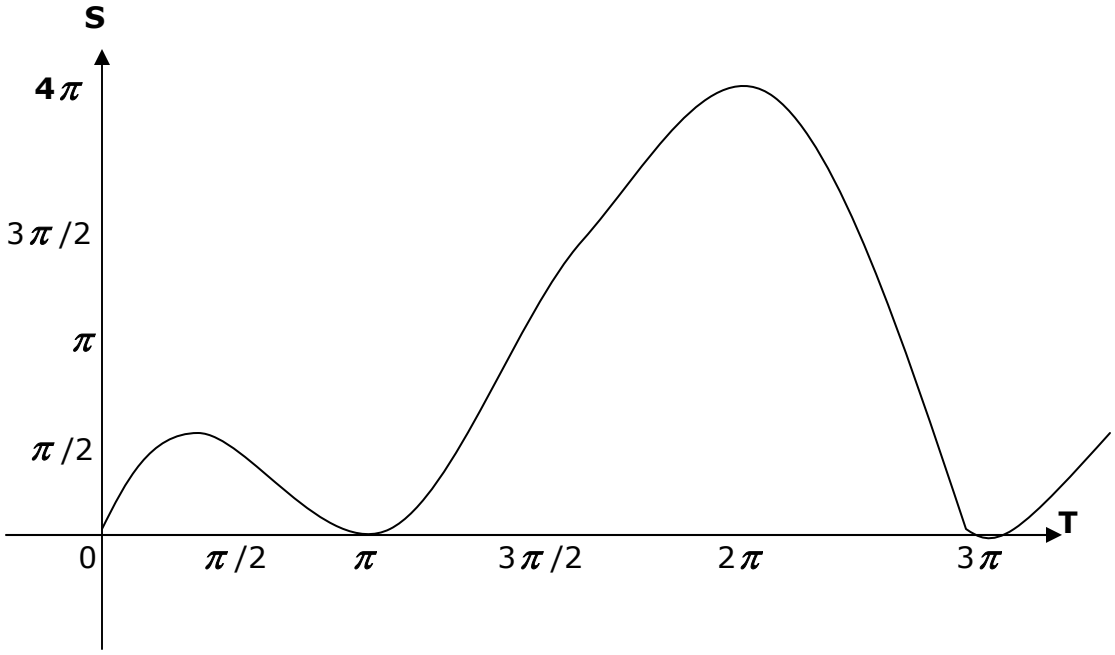
(a) Produce a graph of S against T. Describe the graph in words. **(5 marks)**
(b) Use an exact method of integration to determine the total sales accumulated when $T = 2\pi$. **(5 marks)**
(c) Use Simpson's rule with 7 ordinates to determine the total accumulated sales when $T = 3\pi$. **(6 marks)**
(d) Produce an expansion of S as a power series in T up to T^7. **(4 marks)**

(a) $S = T\,(1 + \cos T)$

the best and the easiest way to construct trigonometric graph is to setup a table of values in $[0, 2\pi]$:

T	0	$\pi/2$	π	$3\pi/2$	2π	3π
S	0	$\pi/2$	0	$3\pi/2$	4π	0

Labeling the axes will get you 1 mark!



S is oscillating.

b) Total Sales = $\int_0^{2\pi} T(1+\cos T)dT = \int_0^{2\pi} TdT + \int_0^{2\pi} T\cos TdT$

$$\int_0^{2\pi} TdT = \frac{T^2}{2}\Big|_0^{2\pi} = 2\pi^2$$

$\int_0^{2\pi} T\cos TdT$,using integration by parts :

u = T $\Rightarrow$ du = dT and dv = cosT dT $\Rightarrow$ v = sin T

$\int udv = uv - \int vdu \Rightarrow \int T\cos TdT = T\sin T - \int \sin TdT$

= T sinT - $\int \sin TdT$ = TsinT + cosT $\Big|_0^{2\pi}$

= (2π sin2π + cos2π) -(0sin0 + cos 0) = 1 – (0 +1) = **0**

Therefore , $\int_0^{2\pi} T(1+\cos T)dT$ **= $2\pi^2$**

c) Simpson's with 7 ordinates:

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+4f(a+h)+2f(a+2h)+4f(a+3h).....+f(b)\right]$$

a = 0 , b = 3π , h = $\frac{b-a}{n-1} = \frac{\pi}{2}$

$$\int_0^{3\pi} T(1+\cos T)dT \approx$$

$$\frac{\pi}{6}\left[f(0)+4f(\frac{\pi}{2})+2f(\pi)+4f(\frac{3\pi}{2})+2f(2\pi)+4f(\frac{5\pi}{2})+f(3\pi)\right]$$

f(0) = 0(1+cos0) = 0 ; f(π/2) = (π/2)(1+cosπ/2) = π/2 etc….

$\int_0^{3\pi} T(1+\cos T)dT \approx$ **$2\pi^2$**

d) Maclaurin's Expansion : derivative of CosT is –sinT ;Derivative of sinT is cosT.

$$f(x) = f(0) + \frac{x}{1!}f'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + ...$$

f(0) = 0 ; f'(T) = (1)(1+cosT) + T(-sinT) ; f'(0) = 2
f''(T) = -sinT – sinT – TcosT ; f''(0) = 0 ; f'''(T) = -cosT –cosT –cosT+sinT
f'''(0) = - 3 ; f''''(0) = 0 ,etc…
f(T) = 2T -3T^3/3! +…

f(T) = 2T – T^3/2 + T^5/24 – T^7/720 ……