2004 Zone B

6 In order to study the computer awareness of its personnel a company assessed 500 of its workers. Each worker was tested to see if they were competent with Wordprocessing (W), Spreadsheets (S) or Database Systems (D). It was discovered that amongst the assessed workers there were 250 competent with W, 380 with S and 280 with D. 60 people were particularly capable as they were competent in all three computer skills. The survey further suggested that 190 workers were competent with S and D. A similar total of 190 workers were competent with S and W but as many as 250 were competent with both D and W.

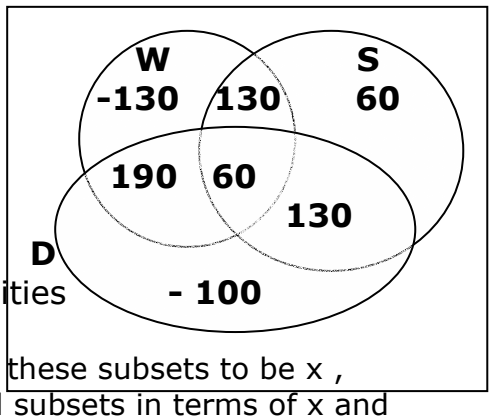
i. Try to produce a Venn diagram for the above situation and hence show that there must be an error in the survey data.

ii. If one (and only one) of the above figures is incorrect determine which ones it could possibly be and, in each case, find a maximum and minimum value which the correct value might be.

iii. Show the following set on a Venn diagram, describe it in words and determine the maximum and minimum order of it:

$$W \cap (D^c \cup S)$$

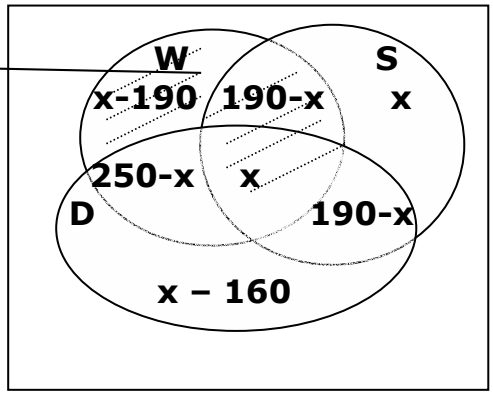
i. It is obvious , in order to make **n(W)** = 250 ;we should have **$n(W \cap S^c \cap D^c) = -130$??**
similarly **$n(D \cap S^c \cap W^c) = -100$??**



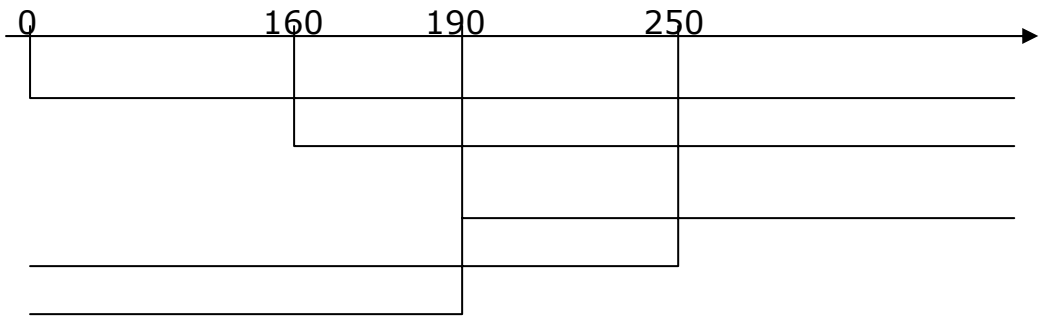
ii. **$n(W \cap S^c \cap D^c)$ and $n(D \cap S^c \cap W^c)$** need to be changed.The only possibilities are **$n(W \cap S \cap D)$ and $n(W \cap D)$**
The strategy is to let the order of one of these subsets to be x , Calculate the orders of all other affected subsets in terms of x and find the limits on the value of x (based on non-negativity of their order).

If **$n(W \cap S \cap D) = x$** is erroneous:

$W \cap (D^c \cup S)$ ← ~~this is for part iii~~

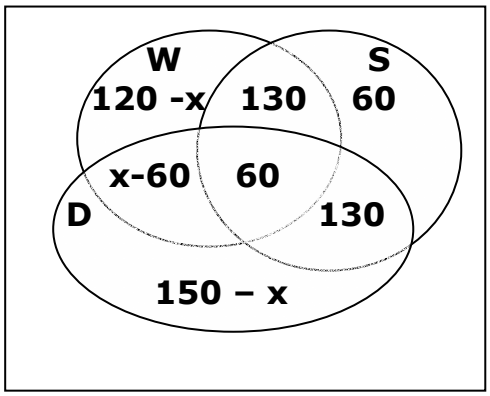


$x - 190 \geq 0 \Rightarrow x \geq 190$; $190 - x \geq 0 \Rightarrow x \leq 190$; $x \geq 0$;
$250 - x \geq 0 \Rightarrow x \leq 250$, $x - 160 \geq 0 \Rightarrow x \geq 160$
putting these on a number line :

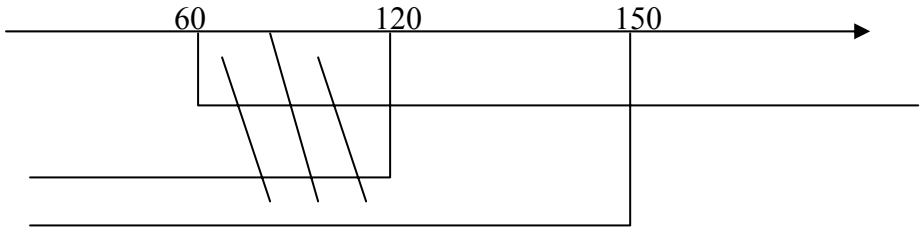


x = 190 is the common solution.

If **n(W∩D) = x** is erroneous:
S∩**W∩D = 60 (remains)**



x – 60 ≥ 0 ⇒ x ≥ 60 ; 120 – x ≥ 0 ⇒ x ≤ 120 ;
150 – x ≥ 0 ⇒ x ≤ 150 ;



60 ≤ x ≤ 120

iii. The brackets **are** important. In words…this subset represents 'Those workers competent with Wordprocessing but not with Wordprocessing and data systems only'. Other, equivalent, expressions are of course allowable. Little words like 'only' and 'but not' are very important – sometimes omitting them drastically changes the statement's logic. Following on from ii., there will be two possible answers for the order of the

W∩(D^c ∪ S)

Following from ii : If the original error was in **(W∩S∩D) ,**from the figure above **n(W∩(D^c ∪ S)) = x – 190 + 190 – x + x = x =** 190
If the error was in **(W∩D) ,**from the **last** figure **:**
n(W∩(D^c ∪ S)) = 120 – x + 130 +60 = 310 - x
but 60 ≤ x ≤ 120 ⇒ -120 ≤ - x ≤ - 60(add 310 to both sides) :
310 – 120 ≤ 310 – x ≤ 310 – 60 ⇒ 190 ≤ 310 – x ≤ 250
190 ≤ n(W∩(D^c ∪ S)) ≤ 250.