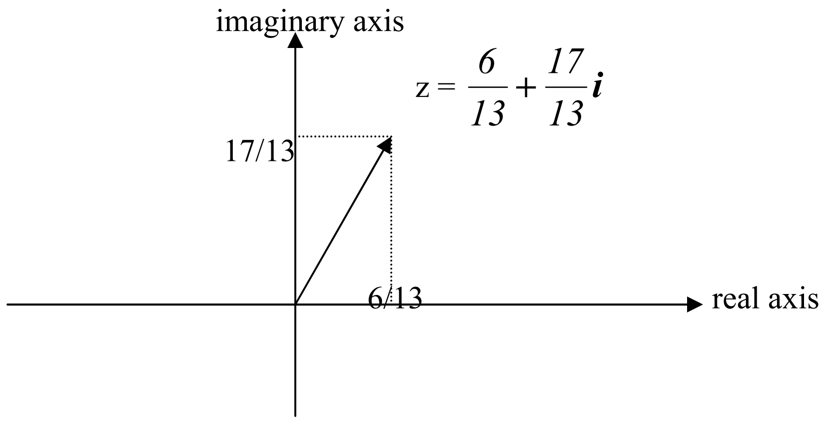
Q5c 2003 ZoneA

c) If $i = \sqrt{-1}$, find the real and imaginary parts of

i) $\dfrac{4+3i}{3-2i}$ ii) $\log_e\left[\dfrac{1}{2}\left(1-i\sqrt{3}\right)\right]$

and draw an Argand diagram for your answer to i). (7 marks)

i. $\dfrac{4+3\boldsymbol{i}}{3-2\boldsymbol{i}} \times \dfrac{3+2\boldsymbol{i}}{3+2\boldsymbol{i}} = \dfrac{6+17\boldsymbol{i}}{13} = \dfrac{6}{13} + \dfrac{17}{13}\boldsymbol{i}$



ii. Use Euler's identity : **r** $e^{i\theta} = r(\cos\theta + \boldsymbol{i}\sin\theta)$

Now you need to convert $\dfrac{1}{2}\left(1-\boldsymbol{i}\sqrt{3}\right) = \dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\boldsymbol{i}$ to trigonometric form.

z = a + ib is converted to z = r(cosθ + isinθ) where

r = $\sqrt{a^2+b^2}$ and θ = $\tan^{-1}\dfrac{b}{a}$

For $\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\boldsymbol{i}$: r = $\sqrt{\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{\sqrt{3}}{2}\right)^2} = 1$; θ = $\tan^{-1}(-\sqrt{3})$ = -60°

(your calculator mode in degrees:

Now convert this to radians : $-60 \times \dfrac{\pi}{180} = \dfrac{-\pi}{3}$ rd

(you may **add** 2π to get rid of the –ve sign: $\theta = \dfrac{5\pi}{3}$)

Hence : $\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\boldsymbol{i} = e^{i\frac{5\pi}{3}} \Rightarrow \text{Ln}\left(\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\boldsymbol{i}\right) = \text{Ln}\, e^{i\frac{5\pi}{3}} = i\dfrac{5\pi}{3}$