1a). The 120 employees of Union City Manufacturing are classified according to whether or not they are:

Skilled (S) or unskilled

Female (F) or male

Employed on the production line (P) or not

The following table gives the number of employees which fall into each group identified, and also the percentage of the total salary bill paid to each group.

Group	Number of workforce	% of total salary bill
F	57	4 3
P	70	67
S	18	20
F∩P	21	26
F∩S	7	8
S∩P	8	4
F∩P∩S	X (unknown)	Y (unknown)

i) From this table calculate the number of people (as a function of X) in each of the eight disjoint subsets which can be logically identified and produce an appropriate Venn diagram. Similarly produce a fully annotated Venn diagram for each group's % of total salary bill with subset orders as a function of Y. (6 marks)

ii) Assuming that each subset of the above Venn diagrams has **positive** order determine the smallest possible value for X and the largest possible value for Y.

(4 marks)

iii) Assuming the values of X and Y determined in ii), which one of the eight subsets has the lowest salary per person? (3 marks)

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Venn diagram - WorkForce
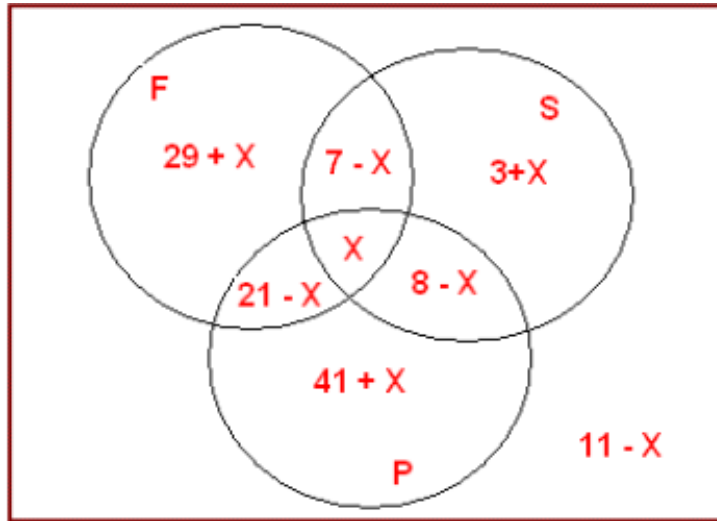
Venn diagram - % of total salary bill
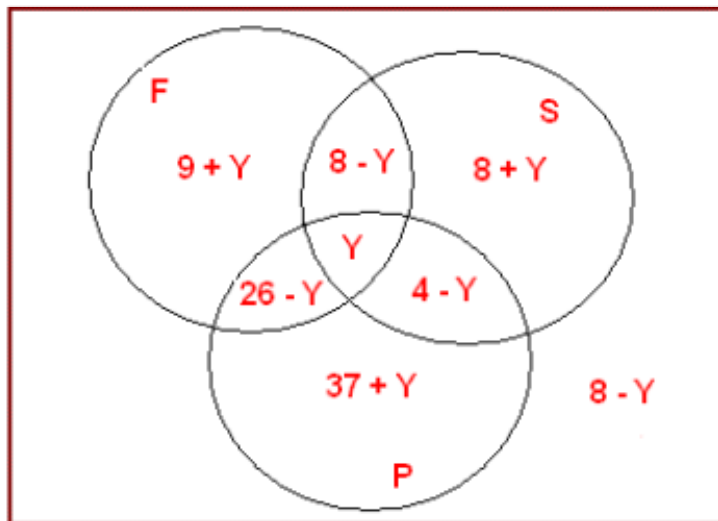
ii. Smallest value of X

Since the order of each subset is positive :

X > 0
29 + X > 0 ; X > - 29
7 - X > 0 ; X < 7
3 + X > 0 ; X > -3
8 - X > 0 ; X < 8
41 + X > 0 ; X > -41
11 - X > 0 ; X < 11

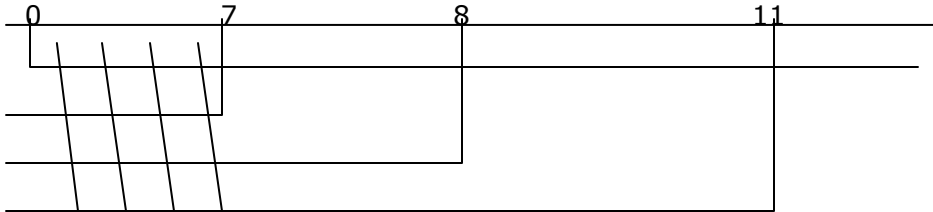Now ignore the inequalities with negative values :

X > 0 , X < 7 , X < 8 , X < 11 implies :

X < 7 and at the same time **X > 0**

i.e. 0 < X < 7
NB : you may see this directly if you put the values on a number line:

Therefore **X = 1,2,3,4,5 or 6** of which **X = 1** is the smallest .

Largest value of Y

In a similar argument as above :
Y > 0 ; Y < 8 ; Y < 4 ; Y < 26
i.e. 0 < Y < 4

Therefore **Y = 1,2 or 3** of which **Y = 3** is the largest .

iii. Lowest salary per person

All you need here is to substitute **X = 1 , Y = 3** in the eight subsets ,then divide the salary bill by the number of the workforce ;the following table shows that the least group with salary bill lies in **S ∩ P ∩ F^c :**

Subset	Workforce	Value	Salary bill	Value	Ratio
F∩P^c∩S^c	29 + X	30	9+Y	12	12/30 = 0.4
F∩S∩P^c	7 − X	6	8 − Y	5	5/6 = 0.83
S∩F^c∩P^c	3 + X	4	8 + Y	11	11/4 = 2.75
P∩S∩F^c	8 − X	7	4 − Y	1	1/7 = 0.14
P∩F^c∩S^c	41 + X	42	37 + Y	40	40/42 =0.95
F∩P∩S^c	21 − X	20	26 − Y	23	23/20 =1.15
F^c∩P^c∩S^c	11 - X	10	8 − Y	5	5/10 = 0.5
F∩S∩P	X	1	Y	3	3/1 = 3