International Institute for Technology and Management

November 28th ,2005

Tutoring Sheet #6

Unit 76: Management Mathematics –Differential Equations

1. Solve the following Differential Equations:

a. $y'' = xe^x$

b. $2\sqrt{x}\frac{dy}{dx} = x^2 - 1$

c. $(2x + 3y)dx + (y - x)dy = 0$

d. $y\frac{dy}{dx} = \sqrt{y^2 + 1}$

e. $x^3\,dx + (y+1)^2\,dy = 0$

f. $\frac{dy}{dx} = \frac{y(x+2y)}{x(2x+y)}$

2. Solve the following differential equations:

a. $\frac{dy}{dx} + 2y = 8x^2 - 2$

b. $x\frac{dy}{dx} + 3y = 2x + 5$

c. $x^2\frac{dy}{dx} + xy + y = 0$

d. $\frac{dy}{dx} = (x+y)^2$

e. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 3x^2 + x + 2$ If $y = 1$ and $\frac{dy}{dx} = 1$ when $x = 0$

f. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^{2x}$

g. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - y = \sin x$

h. $\frac{d^2y}{dx^2} + y = \sin x + \cos x$

i. $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = e^{2x}\cos x$

j. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 8(x^2 + \sin 2x)$

k. $\frac{d^2y}{dx^2} + 4y = \cos 2x + \cos 4x$

LSE Previous Papers

3. Suppose the consumer demand for a company's only product line depends upon the price according to the following formula:

$$q = 50 - 40p - 7\frac{dp}{dt} + \frac{d^2p}{dt^2}$$

and the supply function q = - 10 + 20p

i) Determine the equilibrium price and quantity if p = 5 and $\frac{dp}{dt} = 31$ when t = 0 .

ii) Produce a sketch graph of p against t and describe the behavior of p .

iii) Suggest how the above model might be used in practice Do you foresee any limitations on its use. (LSE 2003)

4. You are given the following differential equation in y :

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = x e^{3x}$$

If y = 1 and $\frac{dy}{dx} = 3$ when x = 0

Solve the above differential equation of y , graph the solution And describe the graph in words.

(LSE 2004)

5. A company maintains its machines every t days and discovers that the overall maintenance costs of the machines, C , are related to t by the following differential equation:

$$t^2\frac{dC}{dt} - (b-1)tC = -ab$$

where a and b are constants and $C = C_0$ when $t = t_0$

i) Derive C as a function of t and the other given constants.

ii) Graph C against t for the case a = 4, b = 2 , $C_0 = 10$ and $t_0 = 1$
(LSE 2005)