International Institute
for Technology and Management

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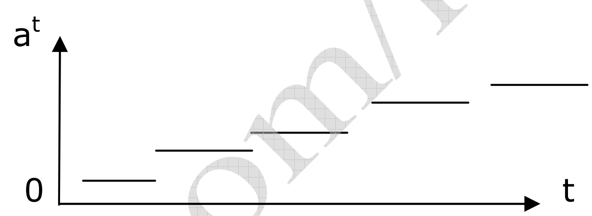
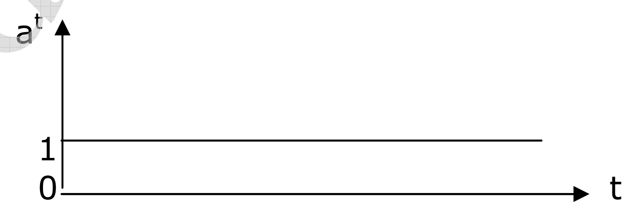
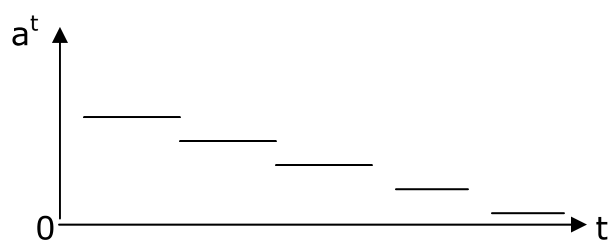
Unit 76:Management Mathematics

Handout #5b

Difference Equations II

Topic	Interpretation
Stability Conditions The general solution of the first order difference equation : $y_t = y_0 + bt$ if a = 1 $y_t = a^t y_0 + b(\frac{1-a^t}{1-a})$ if $a \neq 1$ Can be expressed as : $\mathbf{y_t = y_0 a^t + C}$ $y_c = y_0 a^t$ is called the ***Complementary*** function. $y_p = C$ is called the ***Particular*** solution. i.e. $y_n = y_c + y_p$ y_p expresses the *equilibrium* level of y. y_c represents the *deviations* from that equilibrium. $y_t = y_0 a^t + C$ will be dynamically stable if $a^t \longrightarrow 0$ as $t \longrightarrow \infty$ Depending on the base a, the exponential a^t will generate seven different *time paths*. **In short :** $\|a\| > 1$ time path explodes $\|a\| < 1$ time path converges a > 0 time path nonoscillates a < 0 time path oscillates a = 0 dynamically stable a = 1 dynamically stable a = -1 oscillates between -1 and +1	For simplicity we'll consider the case $y_t = a^t$ ($y_0 = 1$; C = 0)  **a > 1** a^t increases as t increases, thus moving farther away from the horizontal axis. e.g. $y_t = 5(6)^t + 9$,there is no oscillation; with $\|6\| > 1$ the time path explodes.  **a = 1** $a^t = 1$ for all values of t ;dynamically stable.  0 < a < 1 a^t decreases as t increases drawing closer to the horizontal axis but always remaining positive. e.g. $y_t = (1/3)^t$, then as t varies from 0 to 4 : a^t = 1 ,1/3 , 1/9 ,1/27 , 1/81 there is no oscillation; with $\|1/3\| < 1$ the time path converges.

$y_t = y_0 a^t + C$ ($y_0 \neq 1$; $C \neq 0$)
The multiplicative constant y_0 will scale up or down the magnitude of a^t but will not change the basic pattern of the movement.
If $C \neq 0$, the vertical intercept of the graph is affected and the graph shifts up or down accordingly.

Examples :

(1) $y_t = 6(-1/4)^t + 6$

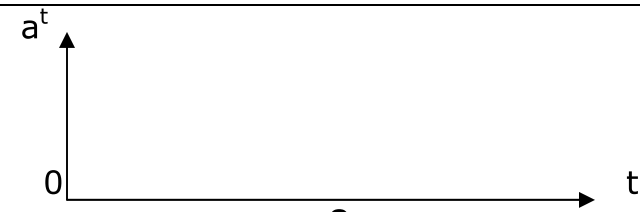
Since $a = -1/4 < 0$, the time path oscillates;
Since $|-1/4| < 1$ it converges.

(2) $y_t = 3(5)^t + 2$

Since $a = 5 > 0$, there is no oscillation;
Since $|5| > 1$, it explodes.

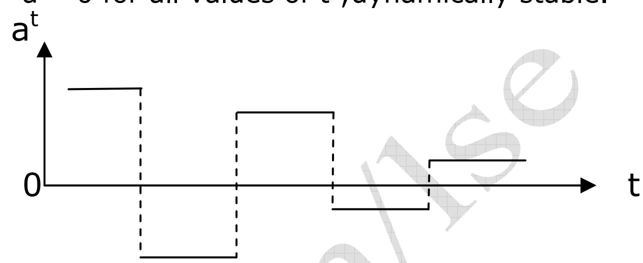
(3) $y_t = (1/8)^t$

Since $a = 1/8 > 0$, there is no oscillation;
Since $|1/8| < 1$, it converges.
Thus the time path is nonoscillating and converging.
(4) $y_t = (-4)^t + 1$
Since $-4 < 0$, oscillates;
Since $|-4| > 1$ it explodes.
Oscillating and exploding.
(5) $y_t = (-1)^t + 5$
Since $a = -1$; $(-1)^t$ oscillates between -1 and + 1, thus y_t oscillates between 4 and 6.



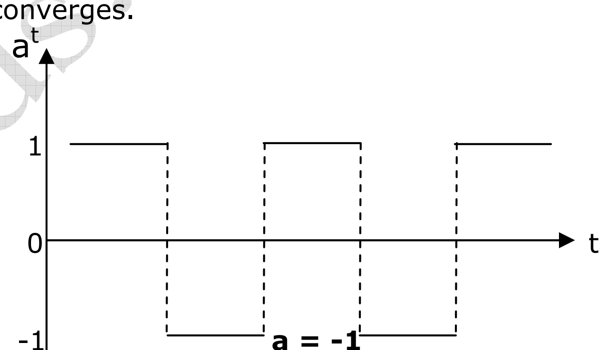
a = 0

$a^t = 0$ for all values of t ;dynamically stable.



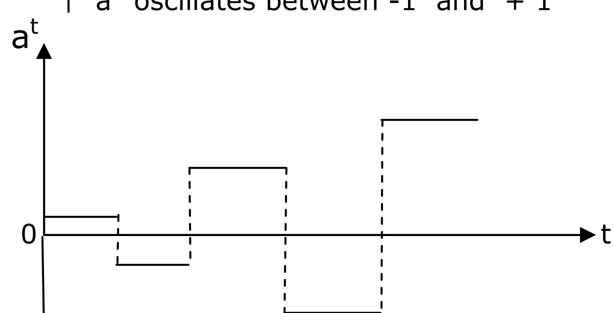
-1< a < 0

a^t will alternate in sign(depending whether t is even or odd) and draw closer to the horizontal axis as t increases.
e.g. $y_t = (-1/3)^t$, then as t varies from 0 to 4 :
$a^t = 1, -1/3, 1/9, -1/27, 1/81$
There is oscillation; with $|-1/3| < 1$ the time path converges.



a = -1

a^t oscillates between -1 and + 1



a < -1

a^t will oscillate and move farther away from the horizontal axis. e.g. $y_t = (-3)^t$, then as t varies from 0 to 4 : $a^t = 1, -3, 9, -27, 81$
there is oscillation;
With $|-3| > 1$ the time path explodes.
Oscillating and exploding.