For comments, corrections, etc…Please contact Ahnaf Abbas: ahnaf@uaemath.com
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International Institute
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Unit 76:Management Mathematics

Handout #5a

Difference Equations I

Topic	Interpretation
Difference Equations Difference equation (or Dynamical system) is a *sequence of numbers* that generated recursively using a rule to relate each number in the sequence to previous numbers in the sequence. <u>Examples:</u> $Y_t = a + bY_{t-1}$; a, b constants. $Y_{k+2} + Y_{k+1} - 6Y_k = 0$ **Order of Difference Equation** The ***order*** of a dynamical system or difference equation is the difference between the largest and the smallest arguments k appearing in it. **Find the order** of the following difference equations: (1) $Y_{k-3} - 3Y_{k-4} = 0$ (2) $Y_{k+3} + aY_{k+1} = bY_{k-1} + c$ (1) k-3 – (k-4) = -3 + 4 = 1	Dynamical system is a system that changes over time. Difference equation is an equation involving differences. We can see difference equation from at least three points of views: as sequence of number, dynamical system and iterated function. x = 5 is an example of a static system because the variable x is constant over time. x = 5t is an example of simple dynamic system. The system is depending on the value of the time t. <u>Example:</u> $Y_{k+1} = a\,Y_k + b$ has order K+1 – k = 1 (2) k+3 –(k-1) = 3 + 1 = 4
Solution of Difference Equations **(1) First order :** We have difference equation $y_k = ay_{k-1} + b$ with initial value y_0 k = 1 : $y_1 = ay_0 + b$ k = 2: $y_2 = ay_1 + b = a[ay_0 + b] + b$ $y_2 = a^2 y_0 + b(1+a)$ ⇒ k = 3 : $y_3 = ay_2 + b$ $= a[a^2 y_0 + b(1+a)] + b$ $y_3 = a^3 y_0 + b(1 + a + a^2)$	k = n : $y_n = ay_{n-1} + b$ $y_n = a[a^{n-1} y_0 + b(1 + a + a^2 + \ldots + a^{n-2})] + b$ $y_n = a^n y_0 + b(1 + a + a^2 + \ldots . a^{n-1})$ $1 + a + a^2 + \ldots . a^{n-1}$ is a GP of first term 1 , number of terms n and common ratio r = a : $1 + a + a^2 + \ldots . a^{n-1} = a_1 \times \frac{1-r^n}{1-r} = 1 \times \frac{1-a^n}{1-a}$ However if a = 1 then $1 + a + a^2 + \ldots . a^{n-1} = n$ $y_n = y_0 + bn$ if a = 1 $y_n = a^n y_0 + b(1 - a^n / 1 - a)$ if $a \neq 1$

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Topic	Interpretation
<u>Examples:</u> (1) $y_k = 2y_{k-1} - 5$; $y_0 = y_0$ (2) $y_k = y_{k-1} + 3$; $y_0 = 7$ **<u>Alternative method:</u>** $y_k = ay_{k-1} + b$ We find a solution which is independent of k;Let this solution be y^* then : $y^* = a y^* + b \Rightarrow y^* = \frac{b}{1-a}$ Now use the substitution: $y_k = y^* + z_k$ $y_k = ay_{k-1} + b$ $\Rightarrow y^* + z_k = a(y^* + z_{k-1}) + b$ $\Rightarrow y^* + z_k = ay^* + az_{k-1} + b$ But $y^* = ay^* + b \Rightarrow z_k = az_{k-1}$ $z_1 = az_0$; $z_2 = az_1 = a(az_0) = a^2 z_0$ $z_k = a^k z_0$ $y_k = y^* + z_k = y^* + a^k z_0$ Since $y_k = y^* + z_k \Rightarrow y_0 = y^* + z_0$ $\Rightarrow z_0 = y_0 - y^*$ Hence $y_k = y^* + a^k (y_0 - y^*)$	(1) $y_n = a^n y_0 + b(1 - a^n / 1 - a)$ a = 2 ; b = - 5 $y_n = 2^n y_0 - 5(\frac{1-2^n}{1-2})$ $y_n = 2^n y_0 + 5(1-2^n) =$ $= 2^n y_0 + 5 - 5 \times 2^n = 5 + 2^n (y_0 - 5)$ (2) a = 1 ; b = 3 ; $y_0 = 7$ $y_n = y_0 + bn$ = 7 + 3n Alternative method solution: (1) $y_k = 2y_{k-1} - 5$ a = 2 ; b = - 5 $y^* = \frac{b}{1-a} = 5$ $y_n = y^* + a^n (y_0 - y^*)$ $y_n = 5 + 2^n (y_0 - 5)$ (2) $y_k = y_{k-1} + 3$; $y_0 = 7$ a = 1 ; b = 3 ; $y_0 = 7$ This says simply that each term is obtained by adding 3 to the previous one $y_1 = y_0 + 3$ $y_2 = y_1 + 3 = y_0 + 2(3)$ $y_3 = y_2 + 3 = y_0 + 2(3) + 3 = y_0 + 3(3)$ $y_n = y_0 + 3n$ with $y_0 = 7$ $y_n = 7 + 3n$