For comments, corrections, etc...Please contact Ahnaf Abbas: ahnaf@mathyards.com

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International Institute for Technology and Management



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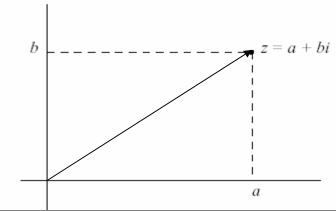
Unit 76: Management Mathematics

Handout #4

Imaginary Numbers

Subject Guide pp 27-29

Algebraic Form: z = a + ib; a = real part; b = imaginary part; $i^2 = -1$ Argand diagram:



Conjugate: z = a + ib, the conjugate z = a - ib;

$$z\bar{z} = a^2 + b^2$$
, for e.g.

$$\frac{2+i}{1+i} = \frac{2+i}{1+i} \frac{1-i}{1-i} = \frac{2-2i+i+1}{1+1} = \frac{3-i}{2} = \frac{3}{2} - \frac{1}{2}i.$$

Magnitude(Modulus): $|z| = \sqrt{z\overline{z}} = \sqrt{a^2 + b^2}$

Polar (trigonometric) Form:

$$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$$
; $r = |z| = \sqrt{x^2 + y^2}$;

 $tan\theta = y/x$; $\theta = Argz$ (argument of z) [Recall Euler's: $e^{i\theta} = cos\theta + isin\theta$]

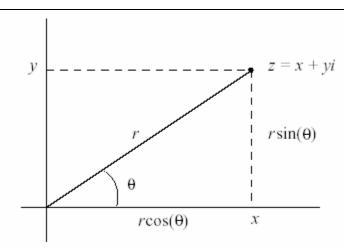
$$\mathbf{r} = \sqrt{2} \; ;$$

$$tan\theta = -1; \theta = -\pi/4;$$

$$z = \sqrt{2} e^{-\pi/4i} = \sqrt{2} [\cos(-\pi/4) + i\sin(-\pi/4)]$$

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Demiovre's

$$(\cos\theta + i\sin\theta)^n = e^{in\theta} = \cos(n\theta) + i\sin(n\theta)$$

$$z = r(\cos\theta + i\sin\theta)$$
; $z^n = r^n(\cos\theta + i\sin\theta)^n = r^n e^{in\theta} = r^n[\cos(n\theta) + i\sin(n\theta)]$
e.g. $(-1 - i\sqrt{3})^6$:

$$r = 2 ; \theta = \pi/3 ;$$

$$[2(\cos(\pi/3) + i\sin(\pi/3))]^6 = 2^6[\cos(\pi/3) + i\sin(\pi/3)] = 64[\cos(2\pi + i\sin(2\pi))]$$

$$= 64[1 + i(0)] = 64$$

Example: Write Ln
$$(\frac{1}{2} - \frac{\sqrt{3}}{2}i)$$
 in the form $a + ib$

Use Euler's identity :
$$\mathbf{r} e^{i\theta} = r(\cos\theta + i\sin\theta)$$

Now you need to convert $\frac{1}{2}(1-i\sqrt{3}) = \frac{1}{2} - \frac{\sqrt{3}}{2}i$ to trigonometric

form.

z = a + ib is converted to $z = r(\cos\theta + i\sin\theta)$ where

$$r = \sqrt{a^2 + b^2}$$
 and $\theta = \tan^{-1} \frac{b}{a}$

For
$$\frac{1}{2} - \frac{\sqrt{3}}{2}i$$
: $r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$; $\theta = \tan^{-1}(-\sqrt{3}) = -60^\circ$

(your calculator mode in degrees:Now convert this to radians : - 60

$$\times \frac{\pi}{180} = \frac{-\pi}{3}$$
rd; you may **add** 2π to get rid of the -ve sign: $\theta = \frac{5\pi}{3}$)

Hence:
$$\frac{1}{2} - \frac{\sqrt{3}}{2} \mathbf{i} = e^{\mathbf{i} \frac{5\pi}{3}} \Rightarrow \operatorname{Ln}\left(\frac{1}{2} - \frac{\sqrt{3}}{2} \mathbf{i}\right) = \operatorname{Ln} e^{\mathbf{i} \frac{5\pi}{3}} = \mathbf{i} \frac{5\pi}{3}$$