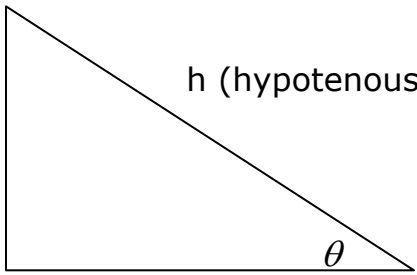




Unit 76: Management Mathematics **Handout #3**

Trigonometric Functions Study Guide pp 23 – 26

Topic	Interpretation																		
<p>Definitions In fig. 3.1 :</p> <p>Sine : $\sin \theta = \frac{o}{h}$</p> <p>Cosine: $\cos \theta = \frac{a}{h}$</p> <p>Tangent: $\tan \theta = \frac{o}{a} = \frac{\sin \theta}{\cos \theta}$</p> <p>Cotangent:</p> <p>$\cot \theta = \frac{a}{o} = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$</p> <p>Secant: $\sec \theta = \frac{h}{a} = \frac{1}{\cos \theta}$</p> <p>Cosecant: $\csc \theta = \frac{h}{o} = \frac{1}{\sin \theta}$</p>	<div style="text-align: center;">  <p>h (hypotenous)</p> <p>(opposite) o</p> <p>a (adjacent)</p> <p>θ</p> <p>fig 3.1</p> </div> <p>Properties $-1 \leq \sin \theta \leq 1$; $-1 \leq \cos \theta \leq 1$ so you will never find an angle θ such that $\cos \theta = 2$. $-\infty < \tan \theta < +\infty$; $-\infty < \cot \theta < +\infty$ It is acceptable to have an angle θ such that $\tan \theta = 200$.</p>																		
<p>Measurements and Periodicity Angles are measured either in degrees ($^\circ$) or in radians (rd)</p> <p>$\alpha^\circ = \alpha_{rd} \times \frac{180}{\pi}$; $\alpha_{rd} = \alpha^\circ \times \frac{\pi}{180}$</p> <p>Periodicity:</p> <p>$\sin \theta$ and $\cos \theta$ are periodic functions of period $2k\pi$ $k \in Z$ (integer) $\cos(\alpha + 2k\pi) = \cos \alpha$; $\sin(\alpha + 2k\pi) = \sin \alpha$</p> <p>$\tan \theta$ and $\cot \theta$ are periodic functions of period $k\pi$ $k \in Z$ (integer) $\tan(\alpha + k\pi) = \tan \alpha$; $\cot(\alpha + k\pi) = \cot \alpha$</p>	<p><u>Example 1:</u> Convert $\frac{5\pi}{6}$ rd into degrees</p> <p>$\frac{5\pi}{6}$ rd $\times \frac{180}{\pi} = \frac{5 \times 180}{6} = 150^\circ$</p> <p>Convert 120° into radians:</p> <p>$120^\circ \times \frac{\pi}{180} = \frac{2\pi}{3}$ rd</p> <table border="1" data-bbox="776 1480 1453 1617"> <thead> <tr> <th>$^\circ$</th> <th>0</th> <th>30</th> <th>45</th> <th>60</th> <th>90</th> <th>180</th> <th>270</th> <th>360</th> </tr> </thead> <tbody> <tr> <th>rd</th> <td>0</td> <td>$\frac{\pi}{6}$</td> <td>$\frac{\pi}{4}$</td> <td>$\frac{\pi}{3}$</td> <td>$\frac{\pi}{2}$</td> <td>π</td> <td>$\frac{3\pi}{2}$</td> <td>2π</td> </tr> </tbody> </table> <p>e.g. $\cos(t + 6\pi) = \cos t$; $\sin(t + 5\pi) = \sin(t + \pi + 4\pi) = \sin(t + \pi) = -\sin t$</p> <p>e.g. $\tan(t + 6\pi) = \tan t$; $\tan(t + 5\pi) = \tan t$</p>	$^\circ$	0	30	45	60	90	180	270	360	rd	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$^\circ$	0	30	45	60	90	180	270	360											
rd	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π											

$\sin(-x) = -\sin(x)$ $\cos(-x) = \cos(x)$ $\tan(-x) = -\tan(x)$ $\cot(-x) = -\cot(x)$ $\sec(-x) = \sec(x)$ $\csc(-x) = -\csc(x)$	$\sin(\pi/2-x) = \cos(x)$, $\cos(\pi/2-x) = \sin(x)$, $\tan(\pi/2-x) = \cot(x)$, $\cot(\pi/2-x) = \tan(x)$, $\sec(\pi/2-x) = \csc(x)$, $\csc(\pi/2-x) = \sec(x)$.	$\sin(\pi/2+x) = \cos(x)$, $\cos(\pi/2+x) = -\sin(x)$, $\tan(\pi/2+x) = -\cot(x)$, $\cot(\pi/2+x) = -\tan(x)$, $\sec(\pi/2+x) = -\csc(x)$, $\csc(\pi/2+x) = \sec(x)$.
$\sin(\pi-x) = \sin(x)$, $\cos(\pi-x) = -\cos(x)$, $\tan(\pi-x) = -\tan(x)$, $\cot(\pi-x) = -\cot(x)$, $\sec(\pi-x) = -\sec(x)$, $\csc(\pi-x) = \csc(x)$.	$\sin(\pi+x) = -\sin(x)$, $\cos(\pi+x) = -\cos(x)$, $\tan(\pi+x) = \tan(x)$, $\cot(\pi+x) = \cot(x)$, $\sec(\pi+x) = -\sec(x)$, $\csc(\pi+x) = -\csc(x)$.	<p>You don't have to memorize these, Use your Calculator to check them. i.e. if you are faced with $\cos(t + \pi)$ choose $t = 30$; find $\cos 30 = 0.86$ and $\cos(30+180) = \cos 210 = -0.86$; hence $\cos(t + 180) = -\cos t$</p>

Basic Relations :

$$\csc \alpha = \frac{1}{\sin \alpha} \qquad \sec \alpha = \frac{1}{\cos \alpha} \qquad \tan \alpha = \frac{1}{\cot \alpha} = \frac{\sin \alpha}{\cos \alpha}$$

$$\sin^2 \alpha + \cos^2 \alpha = 1 \qquad 1 + \tan^2 \alpha = \sec^2 \alpha \qquad \sin \alpha = \pm \sqrt{1 - \cos^2 \alpha}$$

$$\cos \alpha = \pm \sqrt{1 - \sin^2 \alpha}$$

Double Angle Relations

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha = \frac{2 \tan \alpha}{1 + \tan^2 \alpha} \qquad \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \qquad \cot 2\alpha = \frac{\cot^2 \alpha - 1}{2 \cot \alpha}$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 2 \cos^2 \alpha - 1 = 1 - 2 \sin^2 \alpha = \frac{1 - \tan^2 \alpha}{1 + \tan^2 \alpha}$$

Exponential Relations

Where: $i = \sqrt{-1}$; **Euler's Formula :** $e^{i\alpha} = \cos \alpha + i \sin \alpha$

$$\sin \alpha = \frac{e^{i\alpha} - e^{-i\alpha}}{2i} \qquad \cos \alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2}$$

Derivatives:

$y = \sin ax$	$y' = a \cos ax$	$y = \cos ax$	$y' = -a \sin ax$
$y = \tan ax$	$y' = a(1 + \tan^2 ax)$	$y = \cot ax$	$y' = -a(1 + \cot^2 x)$

Integrals:

$$\int \sin ax dx = -\frac{1}{a} \cos ax + c \quad ; \quad \int \cos ax dx = \frac{1}{a} \sin ax + c$$

$$\int \tan ax dx = -\frac{1}{a} \ln (\cos ax) + c \quad ; \quad \int \cot ax dx = \frac{1}{a} \ln (\sin ax) + c$$