International Institute
for Technology and Management

Unit 76: Management Mathematics

Handout #1

Set Theory

Study Guide pp 5 – 11

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Sets

A set is a collection of objects. These objects are called *elements* of the set.
Sets are represented by **A, B, C, etc**...
If A is a set and x is an element of A, we write: $x \in A$.
$x \notin A$ means x is not an element of A.
sets may be described by a common property of its elements rather than by a list of its elements:
{x|x has a property P}
read: the set of all elements x such that x has property P.
A set with no elements is called the *empty* set : ϕ.

Subsets

A set **A** is a *subset* of a set **B** (Written $\mathbf{A} \subseteq \mathbf{B}$) if every element of **A** is also an element of **B**.
For any set **A**:
1. $\phi \subseteq \mathbf{A}$; $\mathbf{A} \subseteq \mathbf{A}$
2. A set of n distinct elements, has 2^n subsets.
e.g. A set of 3 distinct elements has $2^3 = 8$ subsets

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<u>Example1</u>: A = { 5,6,7}

$5 \in A$; $8 \notin A$

<u>Example2:</u>
A={ x|x is a natural number less than 5}
A = { 1,2,3,4}

<u>Example3:</u> the set passengers allowed to smoke in a non smoking flight is an empty set.

0 , ϕ ,{0} should be distinguished:
0: represents a number.
ϕ: represents a set of no elements.
{0}: represents a set with one element. a singleton set.

<u>Example4:</u> **A** is the set of all small businesses with employees less than 20;**B** is the set of all businesses.
Each business with employees less than 20 is also a business, so $\mathbf{A} \subseteq \mathbf{B}$

<u>Example5:</u> List all subsets of { 1,5 ,6 }
There are 8 subsets:
ϕ , {1} , {5} ,{6}
{1,5} ,{1,6} ,{5,6}
{1,5,6}

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Universal set

The universal set in a particular discussion is the set of all objects being discussed.

Venn Diagram

Are used to illustrate relationships among sets. Figure 1.1 shows a set **A** which is subset of a set **B** (**A** is entirely in **B**); the rectangle represents the universal set **U**.

<u>Example6:</u> A company produces only two types of items Large **L** and Small **S**; The universal set here may be denoted by $\mathbf{U} = \mathbf{L} \cup \mathbf{S}$

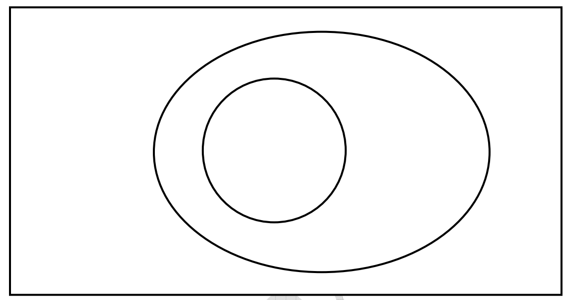


fig 1.1

Operations on sets

Let **A** and **B** be any sets with **U** the universal set then:

1. Compliment $\mathbf{A^c}$:
the *compliment* of set **A** is the set of all elements of **U** which *do not* belong to **A**:

$$\mathbf{A^c = \{x | x \notin A \text{ and } x \in U\}}$$

e.g. If **A** is the set of all female students in a class, then $\mathbf{A^c}$ would be the set of all male students in the class.

2. Intersection $\mathbf{A \cap B}$:
the set of all elements belonging to *both* set **A** ***and*** set **B**:

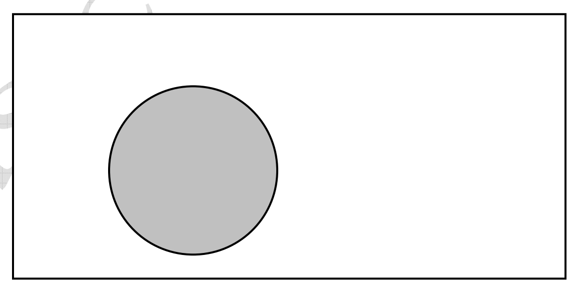
$$\mathbf{A \cap B = \{ x | x \in A \text{ and } x \in B\}}$$

3. Union $\mathbf{A \cup B}$:
the set of all elements belonging to set A **or** to set B **or** to both:

$$\mathbf{A \cup B = \{ x \in A \text{ or } x \in B \text{ or both}\}}$$

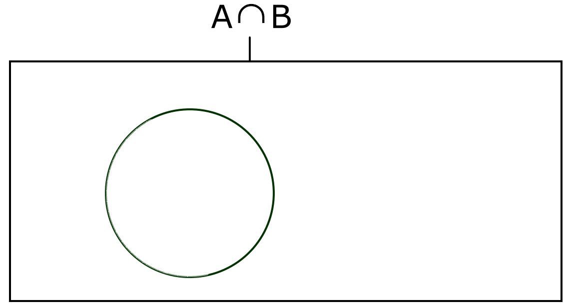
$$\mathbf{A \cup B = \{ 1,3,5,7,4,6\}}$$

U = {1,2,3,4,5,6,7} ; A={1,3,5,7}
B = {3,4,6}



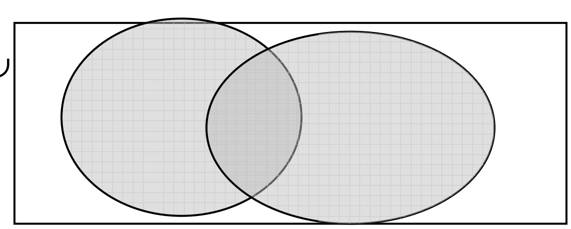
Ac={ 2,4,6} fig 1.2

$A \cap B$



$A \cap B = \{ 3 \}$ fig 1.3

$A \cup$



1.4

4. Related properties :

1. $A \cap A^c = \phi$
2. $A \cup A^c = U$
3. $\phi^c = U$; $U^c = \phi$
4. **Demorgan's Theorems:**
   a. $(A \cap B)^c = A^c \cup B^c$
   b. $(A \cup B)^c = A^c \cap B^c$

5. **Order of a set**

the number of elements in a set **A** is called the *order* of **A**

Written **n(A)** or |A| or n_A .

6 . **Union rule for counting**

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Applications

1. Interpreting statements in set notation:

A good approach is to explain this by examples:

Example1:

Let **M**: the set of all students in IITM taking the management Math. course.

A: all students taking accounting.

S: All students taking statistics.

Interpret each of the following statements in set notation:

a. All students taking management math or accounting or statistics:
   $M \cup A \cup S = U$

   U is the set of all students at IITM serves as Universal.

b. **T**: All students taking accounting and statistics:
   $T \subseteq A \cap S$

c. **N**:All students not taking management math.
   $N \subseteq M^c$

d. **R**:All students not taking accounting and not taking statistics:
   $R \subseteq A^c \cap S^c$

Recall that :

Union means **or**

Intersection means **and**

Compliment means **not**

Example2:

A department store classifies credit applicants by sex , marital status and employment status:

M: the set of male applicants.

S: the set of single applicants.

E: the set of employed applicants.

Describe the following sets in words:

a. $M \cap E$: male **and** employed
   The set of all male employed applicants.

b. $M^c \cap S$: not male **and** single
   The set of all single female applicants.

c. $M^c \cup S^c$: not male **or** not single
   The set of all female or married applicants.

d. $M \cap E = \phi$
   The set of unemployed males.

2. Venn diagrams applications

1. Single set :
Including only a single set **A** inside the universal set, divides **U** into two nonoverlapping regions:

1: represents those elements belonging to set **A**.

2: represents A^c, those elements outside set **A**.

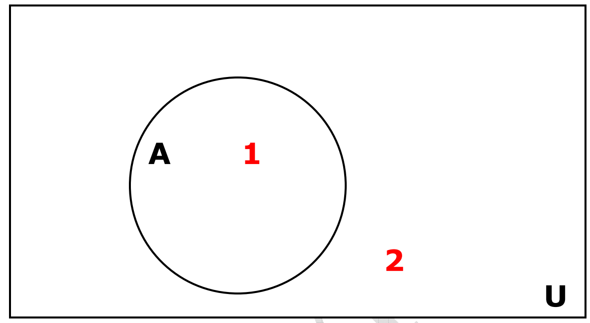


fig 1.5

2. Two sets :
Leads to 4 regions **:**
1: $A^c \cap B^c$:not in **A** and not in **B**
2: $A \cap B^c$: in **A** and not in **B**
3: $A \cap B$: in **A** and in **B**
4: $A^c \cap B$: not in **A** and in **B**

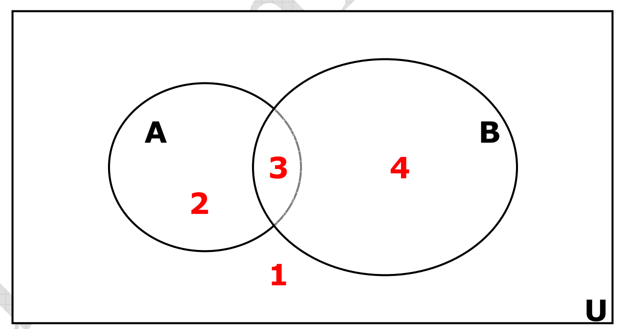


fig 1.6

3. Three sets :
Leads to 8 regions **:**

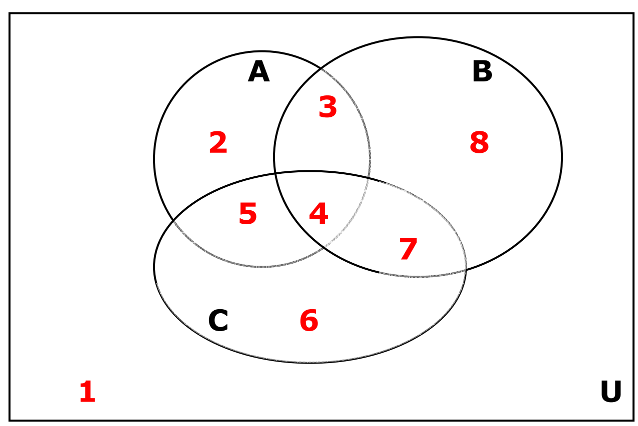


fig 1.7

Example: specify the region for $A^c \cup (B \cap C^c)$: in **B** and not in **C** or not in **A**

Let's find $B \cap C^c$ first :

B is represented by: **3,4,7,8**

C^c is represented by:**1,2,3,8**

The overlapping regions : **3 , 8**
Represents $B \cap C^c$

A^c is represented by **1,6,7,8**

The union of **1,6,7,8** and **3,8**

Is **1,3,6,7,8** which represents $A^c \cup (B \cap C^c)$

Example:
A group of 60 students at IITM was surveyed with the following results:
9 read all three
19 students read Khaleej Times
18 read Gulf Today
50 read Gulf News
13 read KT and GT
11 read GT and GN
13 read KT and GN

a. How many students read none of the publications?

b. How many read **only** GN?

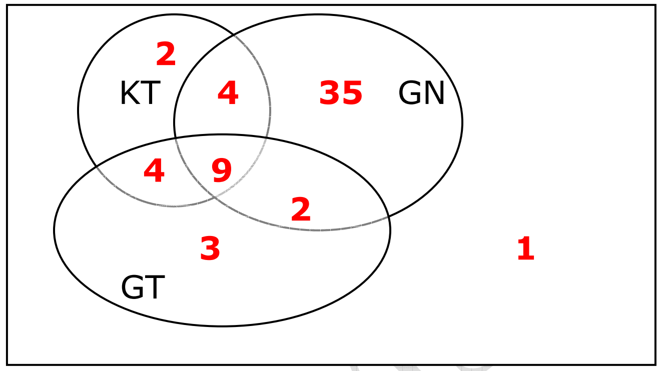
c. How many read KT and GT but not GN?

- Start by placing 9 in the area that belongs to all 3 regions.

-The region representing KT and GT is 13; 9 are allocated so the rest is 4.

- The region representing GT and GN is 11; 9 are allocated so the rest is 2.

-The region representing KT and GN is 13, so the rest is 4.



- The set KT should be 19; but 4+9+4=17 so the rest is 2.

-The set GT should be 18; but 4+9+2=15 so the rest is 3.

-The set GN should be 50; but 4+9+2=15 so the rest is 35.

-A total of: 2+4+3+2+35+4+9 = 59 are placed in various regions. Since 60 are surveyed 60 – 59 = **1 student reads none** of the publications is placed in the other regions.

- **35** read only GN.

- **4** read KT and GT but not GN.