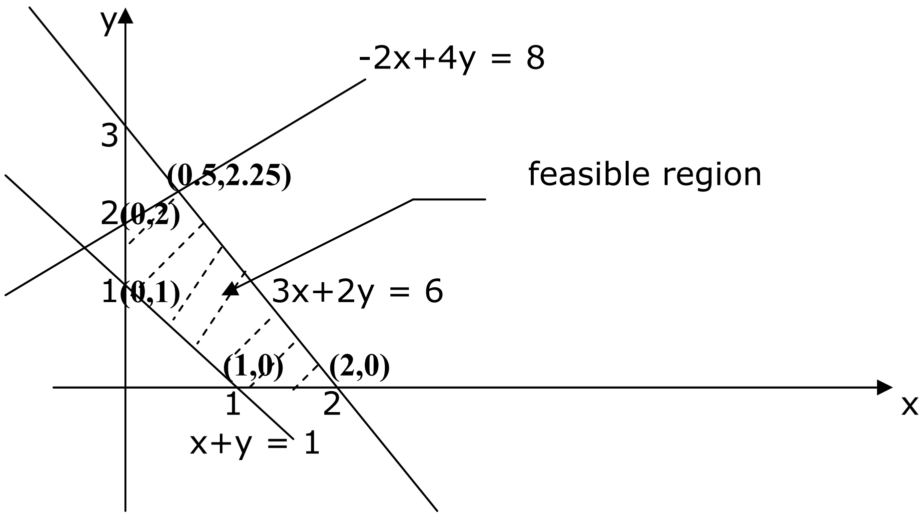
Linear Programming: Graphical method

Example : Find the maximum and the minimum value of **2x + 5y**
subject to: $3x + 2y \leq 6$; $-2x + 4y \leq 8$; $x + y \geq 1$
$x \geq 0$, $y \geq 0$

<u>Step 1</u> : Construct the graphs of the straight lines :
$3x + 2y = 6$; $-2x+4y = 8$, $x+y = 1$, $x = 0$ (y-axis), $y=0$ (x-axis)
The region bounded by these straight lines is called the **feasible region**:



<u>Step 2</u> : Identify the **corner** points, corner points are the points of intersection between the straight lines:
for example : $-2x + 4y = 8$ and $3x + 2y = 6$: solve simultaneously to get $x = 0.5$ and $y = 2.25$.

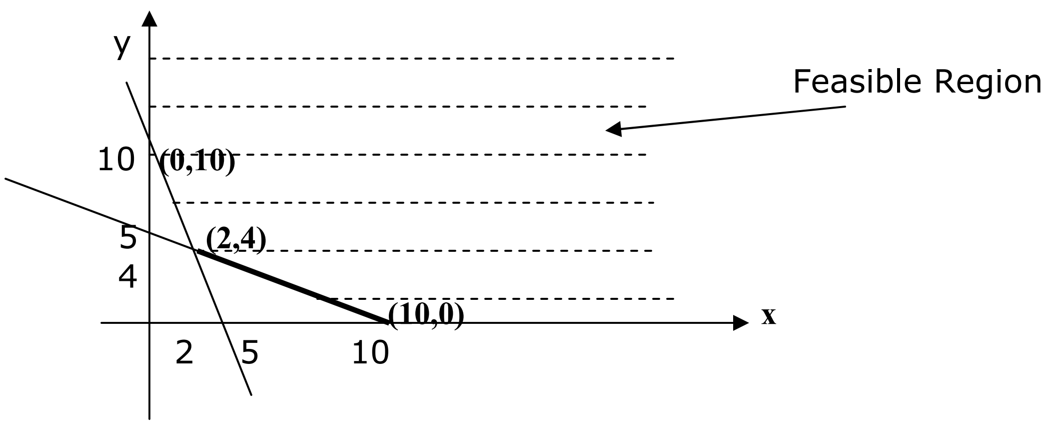
<u>Step 3</u> : Substitute the corner points in the objective function to get the optimum(max and min) values:

Corner point	Value of 2x + 5y
(0 ,1)	2(0) + 5(1) = 5
(0,2)	2(0) + 5(2) = 10
(0.5,2.25)	2(0.5) + 5(22.5) = **12.25 (maximum)**
(2,0)	2(2) + 5(0) = 4
(1,0)	2(1) + 5(0) = **2 (minimum)**

Hence $x = 1$, $y = 0$ minimizes 2x+5y and the value of the minimum is 2.
$x =0.5$, $y = 2.25$ maximizes 2x+5y and the value of the maximum is 12.25.

The Corner point theorem :
If the feasible region is **bounded**, then the objective function **has both** a maximum and a minimum value and each occur at one or more corner points.

If the feasible region is **unbounded** ,the objective function may not have a maximum or a minimum. But if a maximum or minimum value exists, it will occur at one or more corner points.
Example : Minimize **2x + 4y** subject to : $x+2y \geq 10$; $3x + y \geq 10$
$x \geq 0$, $y \geq 0$



Corner point	Value of 2x + 4y
(0,10)	2(0) + 4(10) = 40
(2,4)	2(2) + 4(4) = 20 minimum
(10,0)	2(10) + 4(0) = 20 minimum

Both (2,4) and (10,0) as well as all the points on the line x+2y = 10 lying between them give the same minimum value.There is an infinite number of points minimizing **2x + 4y** subject to the above constraints.

Using Lagrange :
$L = 2x + 4y - \lambda_1(x+2y-10) - \lambda_2(3x+y-10)$

$\frac{\partial L}{\partial x} = 2 - \lambda_1 - 3\lambda_2 = 0$; $\frac{\partial L}{\partial y} = 4 - 2\lambda_1 - \lambda_2 = 0$

$\frac{\partial \boldsymbol{L}}{\partial \lambda_1} = x+2y-10 = 0$; $\frac{\partial \boldsymbol{L}}{\partial \lambda_2} = 3x+y-10 = 0$; solving the last two simultaneously : x = 2 , y = 4.As you see Lagrange gives only one solution. Always use LaGrange. Use Linear programming when you reach a dead end with Lagrange or you are asked to use linear prog.