This paper is not to be removed from the Examination Halls

UNIVERSITY OF LONDON 279 0076 ZA

BSc degrees in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Management Mathematics

Monday, 2 June 2003 : 10.00am to 1.00pm

Candidates should answer **FIVE** of the following **EIGHT** questions. All questions carry equal marks.

Graph paper is provided. If used, it must be securely fastened inside the answer book.

New Cambridge Statistical Tables (second edition) are provided.

A hand held non-programmable calculator may be used when answering questions on this paper. The make and type of machine must be stated clearly on the front of the answer book.

PLEASE TURN OVER

UL03/495

1a). The 100 employees of Vennindex plc company are classified according to whether or not they are: Skilled (S) or unskilled

Male (M) or female

Employed on the production line (P) or not

The following table gives the number of employees which fall into each group identified, and also the percentage of the total salary bill paid to each group.

Group	Number of workforce	% of total salary bill
M	67	63
P	70	87
S	12	13
$M \cap P$	51	56
$M \cap S$	7	8
$S \cap P$	3	4
$M \cap P \cap S$	X (unknown)	Y (unknown)

i) From this table calculate the number of people (as a function of X) in each of the eight disjoint subsets which can be logically identified and produce an appropriate Venn diagram. Similarly produce a fully annotated Venn diagram for each group's % of total salary bill with subset orders as a function of Y. (6 marks)

ii) Assuming that each subset of the above Venn diagrams has **positive** order determine the largest possible value for X and the smallest possible integer value for Y. (4 marks)

iii) Assuming the values of X and Y determined in ii), which one of the eight subsets has the highest salary per person? (3 marks)

(question continues on next page)

b) The same company Vennindex has collected the following data of total salary bill and the number of workers over six years:

Year	1997	1998	1999	2000	2001	2002
Number of Workers	50	65	55	80	85	100
Total Salary Bill (in millions of £)	1.2	1.4	1.3	2.0	2.5	2.8

i) Determine an index series (Base 2000 = 100) for Salary per person employed by Vennindex for 1997-2002. (4 marks)

ii) How useful is the above index for determining whether Vennindex was paying its workforce more generously in some years than in others? (3 marks)

2. Suppose the consumer demand for a company's only product line depends upon the price according to the formula

$$q = 50 - 40p - 7\frac{dp}{dt} + \frac{d^2p}{dt^2}$$

and that the supply function is

$$q = -10 + 20p$$

i) Determine the equilibrium price and quantity if $p = 5$ and $dp/dt = 31$ when $t = 0$. (12 marks)

ii) Produce a sketch graph of p against t and describe the behaviour of p. (4 marks)

iii) Suggest how the above model might be used in practice. Do you foresee any limitations on its use? (4 marks)

3a). The following matrix $\{F_{ij} \mid i,j = \text{A,B...G}\}$ depicts the number of rail passengers travelling

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{pmatrix} 0 & 100 & 200 & 0 & 0 & 0 & 0 \\ 200 & 0 & 80 & 0 & 0 & 0 & 0 \\ 100 & 180 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 120 \\ 0 & 0 & 100 & 20 & 0 & 60 & 0 \\ 0 & 0 & 0 & 10 & 70 & 0 & 30 \\ 0 & 0 & 0 & 100 & 0 & 50 & 0 \end{pmatrix} \end{array}$$

on a particular day between stations A,B,C,D,E,F and G on a railway network.

i) Draw a network flow diagram showing the passenger flow between the stations.

(4 marks)

ii) Another matrix $\{M_{ij} \mid i,j = \text{A,B...G}\}$ depicts the fare for travelling between i and j where

$$M_{ij} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{array} \begin{array}{c} \begin{array}{ccccccc} A & B & C & D & E & F & G \end{array} \\ \begin{pmatrix} 0 & 60 & 20 & 0 & 0 & 0 & 0 \\ 50 & 0 & 50 & 0 & 0 & 0 & 0 \\ 10 & 50 & 0 & 0 & 30 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 130 & 50 \\ 0 & 0 & 30 & 90 & 0 & 10 & 0 \\ 0 & 0 & 0 & 120 & 10 & 0 & 10 \\ 0 & 0 & 0 & 70 & 0 & 15 & 0 \end{pmatrix} \end{array}$$

Assuming each stage of a journey has to be paid for separately, use matrix algebra to determine the total return fares for travelling directly from i to j and back again.

(4 marks)

(question continues on next page)

3b)i) Describe and solve the general 'Gambler's Ruin' problem. (9 marks)

ii) Assume that:

Each player bets and stands to win \$10 on each play;

Initially player A has \$40 and player B has \$30;

That the odds are biased such that for each play A will win with probability 0.6 and B will win with probability 0.4, and

That the game continues until one of the players is ruined.

What is the probability that A will eventually be ruined? (3 marks)

4a) Briefly explain the five stages a modeller might go through in producing an econometric model of a company's sales revenue. (5 marks)

b) The annual sales revenue for a company between 1990 and 2002 is shown in the following table. (The figures are all in millions of £).

Year	90	91	92	93	94	95	96	97	98	99	00	01	02
Sales Revenue	8.3	8.2	9.7	7.0	9.2	7.7	8.5	8.9	10.5	13.6	15.0	17.5	14.6

i) Using a 5-point simple moving average, produce estimates of the Sales Revenue for each of the years 1995 to 2003. (4 marks)

ii) Compare these estimates with those obtained using exponential smoothing using a smoothing constant of 0.3. (5 marks)

iii) Comment upon the two forecasting models used and suggest how you would assess which is the best model. (3 marks)

iv) Suggest a suitable regression model for forecasting sales revenue. (3 marks)

5a)i) State Taylor's theorem for expanding $f(x)$ as a power series, and use it to create expansions for e^{-x} and $\cos x$. (6 marks)

b) Using your answers to i), or otherwise, expand $e^{1-\cos(x)}$ up to terms in x^6 and hence evaluate $\int_0^{\pi/6} e^{1-\cos(x)}\,dx$ (7 marks)

c) If $i = \sqrt{-1}$, find the real and imaginary parts of

i) $\dfrac{4+3i}{3-2i}$ ii) $\log_e\left[\dfrac{1}{2}\left(1-i\sqrt{3}\right)\right]$

and draw an Argand diagram for your answer to i). (7 marks)

6a) For a particular product the daily sales figure for sales, y, follows the following *logistic growth curve*:

$$y = \frac{20{,}000}{1+10e^{-0.05t}}$$

where t is the time in months since the product was launched.

i) Draw a sketch graph of y against t. (4 marks)

ii) Use Simpson's rule with 7 ordinates to evaluate the total sales in the first 6 months after the product's launch. (6 months)

b) If the demand function is

$$P = 20 - Q - \frac{Q^2}{6}$$

and the supply function is

$$5P = Q^2 + 4$$

find the consumers' surplus and producers' surplus at the market equilibrium

(10 marks)

7. An industrial company negotiates contracts which may run for up to 3 months or be cancelled at 1 week's notice. At the end of any given month, t, there are $x(t)$ contracts less than 1 month old, $y(t)$ contracts between 1 and 2 months old and $z(t)$ contracts between 2 and 3 months old.

New contracts arise and some of the existing ones either finish or are cancelled in such a way that the age and number of contracts at the end of one month are dependent upon the age and number at the end of the previous month. The following matrix equation best fits the situation:

$$\begin{pmatrix} x(t+1) & y(t+1) & z(t+1) \end{pmatrix} = \begin{pmatrix} x(t) & y(t) & z(t) \end{pmatrix} P$$

where P is known as the *projection matrix* and is defined to be :

$$P = \begin{pmatrix} 0.2 & 0.9 & 0 \\ 0.5 & 0 & 0.8 \\ 0.3 & 0.1 & 0.2 \end{pmatrix}$$

i) Show that long-run equilibrium values for x, y and z are possible. (4 marks)

ii) If $x(0) = 112$, $y(0) = 80$ and $z(0) = 56$ determine the number of contracts in each age category at the end of each of the next two months. (6 marks)

iii) By inverting the matrix P determine $x(t)$, $y(t)$ and $z(t)$ if $x(t+1) = 13$, $y(t+1) = 16$ and $z(t+1) = 9$. (10 marks)

8a)i) A partitioning (non-hierarchical clustering) algorithm has identified the centres of three clusters, defined by three variables, given below.

		Variable		
		A	**B**	**C**
	1	3.0	3.5	4.4
Cluster	**2**	7.0	4.0	1.5
	3	4.0	1.8	2.5

A new observation (2, 1, 3) occurs. If the distance between two points (a_1, b_1, c_1) and (a_2, b_2, c_2) is $\sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2 + (c_1 - c_2)^2}$, to which cluster should the new observation be allocated and why? (4 marks)

ii) If the distance between two points is $|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2|$, to which cluster should the new observation be allocated and why? (4 marks)

b) The following table gives the distance between cities 5 cities A,B,C,D and E which are to be grouped into two clusters:

	Distance Between Cities			
City	B	C	D	E
A	20	50	80	110
B		40	60	100
C			35	60
D				30

Use the above data to demonstrate the way in which single and complete linkage can lead to different clustering in hierarchical clustering. (12 marks)

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