76 Management mathematics
Examiners' report 2005
Zone A

General remarks

There was no change to the format of this year's examination papers. If it had any bias at all it was towards a little more discussion and comment than some previous year's papers might have contained. However the understanding of the practical uses of the mathematics has always been a key aspect of the unit.

In general the grades achieved by the candidates were rather disappointing. As always there were two or three aspects of the paper that proved to be surprisingly difficult for the students and we offer comments on these in this report – see, for example, Q1(a), Q1(b)i., Q1(b)iii., Q3(a), Q3(d), Q4(b), Q6(a)iii.

There was a need for more candidates to be able to comment sensibly on their results and the mathematical models involved. The examiners were pleased to see that the great majority of candidates tackled 5 full questions and that all questions received a reasonably fair share of attempts.

Specific comments on questions

Question 1

(a) This was not as well done as expected. A good answer would explain what is meant by each of these index types and to give examples of each. It would have also been good to see a statement as to when each might be used. See page 13 in the subject guide.

(b) i. This was not well done with most candidates concentrating entirely upon issues of base changes. To improve this, candidates needed to be specific in their comments. The indices are concerned with agricultural production and hence the answers should recognise this. A good attempt would have discussed the problems of data collection for such a huge economic function, problems of combining the various products (e.g. do we use calories, tonnage, $ value??), problems of whether to collect the data once a year or when the products come to 'harvest' and what sort of index to use.

ii. When combining two indices with different base periods one should use the most recent of the two bases as the base for the new combined index. This should be adhered to unless there is a strong reason to the contrary. Hence in this part of the question the combined production index should have had a base of 2000=100. The index values for 1997 to 1999 are 66.74, 72.09 and 79.62 respectively. For years 2000 to 2004 the values remain the same as for the 'FarmFood' index.

iii. The correct approach to the right index is made up of two steps as follows. Firstly, take the combined Agricultural Production index (from ii. above) and divide the values (year by year) by the corresponding values of the Population. Since we are eventually going to get an index, there was no need to worry about the units and so the fact that the figures for population are in millions could be ignored. Secondly, take the resulting values and choose an appropriate base to rescale them. Thus, for example, the values for Agricultural Production over Population were 66.74/0.92 = 72.54 in 1997 and 100.0/1.11 = 90.09 in 2000. Making 2000 as the best base for the required index (remember the remarks in ii. for choice of base) then the index value for Agricultural Production per head of Population for 1997 was (72.54/90.09) x 100 = 80.52. Similarly for the other years.

Finally don't forget to give some comments upon both the validity and interpretation of the index you have created. Comments were expected to cover the general pattern of index values, when they were highest, when lowest, in which year was the change the most, etc. The index was not particularly valid as a measure since there will be imports and exports, varying age distribution and food requirements of the population. In addition the population will have been difficult to determine as well as the problems noted in i. above.

Overall Question 1 was not particularly onerous – past questions have often involved many more calculations.

Question 2

(a) As with all questions – but especially with this one - there was a need to read the given information carefully. It would probably be beneficial for many candidates to have re-read through the question after they have created their matrix, in order to check that all given information was satisfied. Furthermore, and fundamentally, candidates should remember that the each row of the transition matrix must add up to 1. The required transition matrix, *A* say, was:

$$A = \begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{3}{10} & \frac{4}{10} & \frac{3}{10} & 0 & 0 \\ 0 & \frac{3}{10} & \frac{4}{10} & \frac{3}{10} & 0 \\ 0 & 0 & \frac{3}{10} & \frac{4}{10} & \frac{3}{10} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 \end{pmatrix}$$

(b) The basic methodology was to solve $\pi = \pi A$ for the vector of equilibrium state probabilities $\pi = (\pi_{-1}\ \pi_0\ \pi_1\ \pi_2\ \pi_3)$. A logical approach was necessary and with care the answer for π was obtainable. The solution process would have been simplified if you spotted that, for this transition problem, $\pi_{-1} = \pi_3$ and $\pi_0 = \pi_2$ by symmetry. For most candidates, it would seem most straightforward to have written out the set of 5 linear equations governed by $\pi = \pi A$ and then use substitution/elimination to obtain each π_i in terms of π_* for some * = -1,0,1, 2 or 3. Finally had to use the fact that $\pi_{-1} + \pi_0 + \pi_1 + \pi_2 + \pi_3 = 1$ (candidates earned a mark for this fact even when many other aspects of their answer were falling apart) and to replace each π_i in terms of π_* so that we can solve for π_* and hence each individual π_i.

(c) This was well understood. Candidates obtained clearer diagrams if they maintained the symmetry in the situation (i.e. use the five states in a straight line or, better, in a regular pentagon format) rather than having the five states 'all over the place'.

(d) A connectivity matrix contains elements of 0 (where no direct connection exists) or 1 (where a direct connection exist) between states. This matrix, let us denote it by M, was given by

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

The required answer for the number of ways of moving between states in exactly two steps (i.e. visiting one intermediate state) was given by M^2 i.e. $M.M$. Many candidates created M^3 which was the answer for the number of different routes between states visiting 2 intermediate states i.e. in 3 steps. However, as the term 'step' was not precisely defined in the question, either of the answers M^2 or M^3 were allowed (generously). The expected answer M^2 was

$$M^2 = \begin{pmatrix} 2 & 3 & 4 & 3 & 1 \\ 1 & 3 & 3 & 2 & 1 \\ 1 & 2 & 3 & 2 & 1 \\ 1 & 2 & 3 & 3 & 1 \\ 1 & 3 & 4 & 3 & 2 \end{pmatrix}$$

Question 3

This question caused some difficulties. There was such a choice of alternative correct answers it was surprising as well as disappointing to see the even larger number of incorrect answers that were produced.

(a) The above comments are no more relevant than in this part. For example in ii) one could give any of the following answers: $O \subseteq W^c$ or $O \cap W = \phi$ or $W \subseteq O^c$ etc., BUT NOT $O \in W^c$ or $n(O \cap W) = \phi$ or $O \cap W^c = 0$ etc., etc. Notation (and correct notation) was a key point of this question. Leaving aside notation, the part which caused the greatest difficulty was part iii) where the incorrect, illogical answer $I \subseteq W$ was often seen.

(b) Remember when asked to draw such strange diagrams that one should have it as general as possible whilst, at the same time, not having subsets that we are informed cannot occur. In addition one should try to avoid having any of the defined subsets being split into parts i.e. there should be one contiguous area representing each of the subsets. In essence the Venn diagram should depict each of the above statements, which amount to:

O inside *S*
O separate from *W*
W inside *I*
I separate to *S*
P separate to *O*

Furthermore there should be an area corresponding to "*P* only".

(c) A correct diagram (or the original statements) showed that A) and C) are implied by the given facts but B) was not implied.

(d) Care was needed to avoid ambiguity. For example, for $(S \cup P)^c$ one could describe it as "not in S or P" but this is ambiguous since it could mean "either not in S or not in P" – which would be wrong. Answers that included words like compliment, union, intersection (e.g. 'S union P complement' for the first statement) were immediately marked wrong. The 3 phrases that were suggested in the marking scheme (but obviously not the only ones allowed) were: 'When workforce are not on strike and workforce is not underpaid'; 'When workforce are not on strike or workforce is not underpaid'; 'When workforce strike and workforce are not overpaid and they are not overworked'. Answers that interpreted 'not overpaid' as 'underpaid' or 'price not increasing' as 'price decreasing', etc were (generously) not penalised.

Question 4

This was a popular question which usually earned good marks (but very rarely full marks). Questions like this, which have become quite predictable and remain popular, are now marked quite harshly. Those candidates, for example, who joined locations which were furthest apart, found themselves with very few marks available to them. When the methods were entirely confused or when there was no consistency of approach the marks were reduced considerably. Do take care! One or two marks were also lost by forgetting the particular aims of the question. The majority of candidates, once embarked upon their clustering process, proceeded all the way to the final stage of having only one cluster of all the locations. However the question refers to clustering so as to use three distribution centres to serve the locations – hence the clustering process should have stopped when we had three clusters. The dendograms should include a horizontal scale and avoid crossing over of lines.

The discussion parts of this question were quite tricky, and there was strictly no right and wrong answers. Any well argued case (including some practicality) was given good credit. Vague guesses rarely earned many marks. The preferred answer at the end of part (a) for which method is most appropriate was along the following lines:

> Single linkage is the least appropriate since it may lead to chaining and unbalanced clusters. Complete linkage is a better criteria her, since depots will be situated within the centre of a cluster, all locations of which should be within a maximum distance from the depot.

For part (b) we have a different situation since the distribution centres already exist. It would now seem most sensible to have each location served by its closest distribution centre. Thus *A* will be served by *B*, *D* will be served by the distribution centre at *C*, whilst *F* will be served by the centre at *E*. Distribution centres at *B*, *C* and *E* will serve themselves. Any nicely argued practical approach would have been well rewarded.

Question 5

Part (a)i needed to be recognised as a linear first order, first degree differential equation. Then the standard procedure using an integrating factor (see page 39 of the subject guide) works fine – although care was needed. For those who spotted the approach the errors occurred because of carelessness with signs and a lack of appreciation of the standard form of the equation required in order to apply the integrating factor methodology. In outline, we first rewrite the given equation in the standard form of $\frac{dC}{dt} + PC = Q$ where $P = -\frac{(b-1)}{t^2}$ and $Q = -\frac{ab}{t^2}$.

The integrating factor was then

$$e^{\int Pdx} = e^{-\int \frac{b-1}{t}dt} = e^{-(b-1)\ln t} = e^{\ln(t^{1-b})} = t^{1-b}$$

and multiplying each term in our standard form by this integrating factor and then integrating both sides of the equation gaves us $t^{1-b}C = \int -abt^{-b-1}dt = at^{-b} + k$

Thus $C = \frac{a}{t} + kt^{b-1}$ and, finally, by inserting the initial conditions we found $k = \frac{C_0t_0 - a}{t_0^b}$ and hence $C = \frac{a}{t} + \left(\frac{C_0t_0 - a}{t_0^b}\right)t^{b-1}$.

(a)ii. For $a = 5$, $b = 2$, $C_0 = 20$ and $t_0 = 1$ then $C = (5/t) + 15t$. Probably the simplest mark on the whole paper was earned for labelling the horizontal and vertical axes as t and C respectively – strange how many students failed to get the mark! Since t_0 was 1 rather than zero, then the graph did not go to infinity for the initial value of t – indeed the graph started at $C = 20$. It then diminished to a minimum when $t = \sqrt{(1/3)}$ before growing (more or less linearly with t) thereafter.

(b)i. This should be very straightforward. It is, after all, a common type of economic model. By substituting the appropriate functions of Y for C_t and I_t in the fourth equation we get

$$Y_t - (k + \frac{4}{15})Y_{t-1} + \frac{4}{15}Y_{t-2} = 10 .$$

The auxiliary equation was $m^2 - (k + 4/15)m + 4/15 = 0$ which gave oscillating solutions for Y_t if "$b^2 < 4ac$" i.e. $(k + 4/15)^2 < 16/15$. For positive k, this meant that $k < 0.7661$. Hence Y_t will oscillate if $0<k<0.7661$.
Note: There was no requirement to actually solve the difference equation.

Question 6

(a)i. It is often the initialisation of these forecasting methods that cause most difficulties. Many students fall into the 'trap' of assuming data for a period is known when trying to forecast for that very same period. Thus, in this question we were asked to make a prediction for February, initially, and the only known datum at that stage was the January sales figure of 28. Hence we start the exponential smoothing process with an initial forecast of 28 for February. From that point on we make use of the exponential method $F_t = \alpha X_{t-1} + (1-\alpha)F_{t-1}$ (taking care not to create similar looking – but erroneous – equations) to give the forecasts for March – October.
ii. Once again it is issues about how to start the forecasting process that cause most problems. Specifically for moving averages it is a question of where to position the first forecast. There was no indication in the question that one is using the moving average process as an initial stage in determining a trend and seasonality analysis. There was, however, ever indication that the moving average itself constitutes the forecast. Hence when we have averaged the first 5 values, to get (28 + 22 + 24 + 28 + 30)/5 = 26.40, we position this as the forecast for June's sales. In a similar way we get forecasts for July onwards. To make a comparison of the two forecasting methods using Root Mean Square Error we should, to be fair, take the forecasts over the same period i.e. in this case over the period June to September. There was no way that one can compare Root Mean Square Errors with Mean Absolute Deviations. Remember that this is ROOT mean Square error (the square root was often forgotten).

iii. To improve upon the forecasting methods we could include trend, seasonality, use more data, include other variables (e.g. price/weather/advertising etc.) or investigate using a different exponential smoothing constant or a different number of periods in the moving average.

(b) This was straight from the subject guide (pp. 66-69). Ideally candidates should have said more than just ‘Trend, Seasonality, Cycles, Random Variation’ and have included a sentence about each one. The mark most commonly missed was a statement as to why the approach is so attractive. Again a suitable answer was contained directly in the subject guide i.e. ‘An attraction of the (decomposition) approach is that the components are intuitively reasonable and can be easily explained’.

Question 7

This question was a good test of a candidate’s ability to work with several mathematical concepts.

The nature of *S* is one of growing oscillations. The most difficult feature to identify, perhaps, is that *S* equals zero when $T = 0$, π, 3π etc. ‘*S* oscillates with increasing magnitude and is unstable’ is an adequate description of the graph.
(b) The correct approach was to use integration by parts. Thus, setting
$'u' = T, 'dv/dt' = 1 + \cos T \rightarrow du/dt = 1, v = T + \sin T$
the total sales =

$$\int_0^{2\pi} T(1+\cos T)dT = T(T+\sin T)\Big|_0^{2\pi} - \int_0^{2\pi}(T+\sin T)dT =$$

$$T^2 + T\sin T - \frac{T^2}{2} + \cos T\Big|_0^{2\pi} = 2\pi^2.$$

(c) This was a standard Simpson’s rule type question with the added complication of needing to be able to handle a *cos* function (and to remember to work with radians for such a question since the upper integral limit is given as 3π which was presumable in radians and not degrees). Note that there were seven ordinates requested which would therefore be for $\theta = 0$, π/2, π, 3π/2, 2π, 5π/2 and 3π and hence you will need to evaluate $f(T)$ for these values of T. One final word of warning, this part asked for an integral between 0 and 3π not 0 and 2π as in part (b).

(d) Take special care with the signs and the factorials. Otherwise it is quite straightforward. But do read the question – there is no requirement to integrate anything in this part of the question. The correct answer was

$$S = 2T - \frac{T^3}{2} + \frac{T^5}{24} - \frac{T^7}{720} \ldots\ldots$$

Question 8

Note that in (a)i. you were required to use critical values from a t distribution based upon 12 degrees of freedom (not a Normal distribution which only applies when the number of observations is rather larger. Remember, also, to formally state the null and alternative hypotheses.

ii. This is a matter of opinion and hence a number of well argued cases could have earned the marks. The examiners, however, were really expecting the answer of yes, there is likely to be multicollinearity since growth rate will depend upon current dividend. A statement of what multicollinearity was (see subject guide p. 78) also needed.
iii. Once again some common sense was required. Comments such as; 'Use more data, additional variables (e.g……), drop *G ??*' earned credit.

(b) See the subject guide page 77. Explain the Durbin Watson test.

Key steps to improvement

Too many candidates were giving one-sentence answers for comment questions designated 5 or 6 marks or giving 2 page answers for questions designated 2 or 3 marks. Candidates are advised to be aware of the number of marks awarded to each part of the questions in order to better judge the amount of comments that might be required. The weaker candidates should be encouraged to make sure they squeeze out every possible mark and not to miss the easy ones like labelling the axes of a graph, or giving simple comments etc.

Examination paper for 2006

There will be no change to the format, style or number of questions in the examination paper for 2006.