UNIVERSITY OF LONDON

279 0076 ZA

BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Management Mathematics

Wednesday, 8 June 2005 : 10.00am to 1.00pm

Candidates should answer **FIVE** of the following **EIGHT** questions. All questions carry equal marks.

Graph paper is provided. If used, it must be securely fastened inside the answer book.

New Cambridge Statistical Tables (second edition) are provided.

A hand held non-programmable calculator may be used when answering questions on this paper. The make and type of machine must be stated clearly on the front cover of the answer book.

1.(a) Briefly discuss the differences between Relative and Aggregate Indices. **(4 marks)**

(b) The following table gives a index of agricultural production in the country of Groland using two index methods (The "Foodpro" index based in 1980 and the "FarmFood" index based in 2000). Also given is an estimate of population of Groland.

Year	"Foodpro" index (Base 1970=100)	"FarmFood" index (Base 2000=100)	Groland's Estimated Population (million)
1997	360.6		0.92
1998	389.5		1.05
1999	430.2		1.07
2000	540.3	100.0	1.11
2001		104.5	1.23
2002		120.3	1.31
2003		115.3	1.40
2004		132.1	1.45

   i. Briefly discuss the likely difficulties that there will have been in determining the agricultural production indices? **(4 marks)**

   ii. Combine the two production indices for agricultural production so that the resultant series has a common base. **(4 marks)**

   iii. Produce a fixed base index series for agricultural production per head of population. Comment thoroughly upon the validity and interpretation of your results. **(8 marks)**

2. In a 5 state Markov chain the steps have probabilities 0.3, 0.3 , 0.4 of taking values 1, -1 or 0 respectively. There are reflective barriers at 3 and at -1 and these reflective barriers work such that, if the system reaches a barrier at time n, it is reflected back to any of the other 4 states at time period $n+1$ with equal probability.

(a) Write down the transition matrix of the Markov chain. **(3 marks)**

(b) Determine the equilibrium probabilities of being in the various states. [Note: candidates may find it useful to note some symmetry in the situation.]**(10 marks)**

(c) Draw a suitably annotated network diagram to illustrate the above Markov Chain. **(3 marks)**

(d) Create a connectivity matrix to show which states one can move between in exactly one step and use it to determine the number of ways one can move between states in exactly two steps. **(4 marks)**

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3. As a consequence of daily observations of a company, you discover the following facts about the sets O, S, W, I, P relating to the affairs of the company:

      i)   When the workforce are overworked(O) they always strike(S);

      ii)   The workforce are never overworked during the winter(W);

      iii)   The company's share price always increases(I) during the winter;

      iv)   The company's share price never increases when the workforce are on strike(S).

      v)   The workforce are never underpaid(P) when overworked.

(a) Convert each of the above facts into a mathematical statement about the appropriate sets. **(5 marks)**

(b) Draw a single Venn diagram to illustrate the above relationship between the sets. **(6 marks)**

(c) For each of the following statements indicate whether they are implied by the above facts:

    A)   No strikes occur in the winter

    B)   The share price never increases when the workforce are overpaid

    C)   The workforce never strike when they are underpaid in the winter.

(3 marks)

(d) For each of the following subsets, describe its meaning in words and use a Venn diagram similar to (b) above to depict it:

$$(S \cup P)^c$$
$$(S \cap P)^c$$
$$S \cap P^c \cap O^c$$

(6 marks)

4. A company supplies to departmental stores at 7 locations $A, B, \ldots G$. The following matrix depicts the road distance (in miles) between these locations:

To:

		B	C	D	E	F	G
	A	11	19	39	21	35	12
	B		15	34	26	38	13
From:	C			23	20	42	22
	D				27	24	41
	E					14	30
	F						47

(a) Using   i. single linkage clustering and
                      ii. complete linkage,

produce clusters of locations such that these clusters may be used to decide which locations are served by each of the three distribution centres (not necessary at any of the given locations). Produce a dendogram to illustrate each solution and state which of these linkage methods is usually the most applicable for this sort of exercise. **(15 marks)**

(b) If, conversely, the three distribution centres already exist at locations B, C and E, suggest (with reasons) which location each centre will serve. **(5 marks)**

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5.(a) A company maintains its machines every t days and discovers that the overall maintenance costs of the machines, C, are related to t by the following differential equation:

$$t^2 \frac{dC}{dt} - (b-1)tC = -ab$$

where a and b are constants and $C = C_0$ when $t = t_0$.

  i.   Derive C as a function of t and the other given constants. **(8 marks)**

  ii.   Graph C against t for the case of $a = 5$, $b = 2$, $C_0 = 20$ and $t_0 = 1$ **(4 marks)**

(b) For some positive $0<k<1$, the following equations relate to Consumption C_t, Investment I_t, Income Y_t and Production Q_t at time t:

$$C_t = kY_{t-1}$$

$$I_t = 10 + \frac{1}{3}(Q_{t-1} - Q_{t-2})$$

$$Q_t = \frac{4}{5}Y_t$$

$$Y_t = C_t + I_t$$

  i.   Derive a second order difference equation for Y in terms of k. **(2 marks)**

  ii.   Determine the range of values for k for which Y_t oscillates. **(6 marks)**

6.(a) A supermarket has been recording data for its sales of beer for many years. Its most recent results are given in the following table:

Month	Sales of beer (litres x 1000)
January	28
February	22
March	24
April	28
May	30
June	35
July	38
August	35
September	29

  i.   Using exponential smoothing with a smoothing constant of $\alpha = 0.2$, develop forecasts for February through to October. What is the Mean Absolute Deviation between February and September? **(6 marks)**

  ii.   Develop a five month moving average for the same data and compare the accuracy of the two forecasting methods using Root Mean Square Error over a suitable period. **(5 marks)**

  iii.   How you might improve upon the above forecasting methods. **(4 marks)**

(b) What is meant by a 'decomposition approach' to forecasting a time series ? Why is such an approach so attractive? **(5 marks)**

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7.   The rate of sales (S) of a newly launched product is expected to be given by the following function of time (T) from launch:

$$S = T(1 + \cos T)$$

(a)   Produce a graph of S against T. Describe the graph in words.   **(5 marks)**

(b)   Use an exact method of integration to determine the total sales accumulated when $T = 2\pi$.   **(5 marks)**

(c)   Use Simpson's rule with 7 ordinates to determine the total accumulated sales when $T = 3\pi$.   **(6 marks)**

(d)   Produce an expansion of S as a power series in T up to T^7.   **(4 marks)**

8.(a)   A financial advisor for a large investment company regressed the price of a share (P) on its dividend rate (D) and the expected growth rate in the dividend (G). Data from the last 15 years were recorded and analysed.

Output from the regression package used to fit the equation includes the following information:

Regression Output:

Constant	- 0.210
Std Err of Y Est	0.138
R Squared	0.998
No. of Observations	15
Degrees of Freedom	12

	D	G
Regression Coefficient:	1.922	0.157
Std. Err. of Coefficient:	0.052	0.114

The fitted least squares regression equation is therefore

$$P = - 0.210 + 1.922D + 0.157G + \text{error}$$

i.   Test the significance of each regression coefficient at the 1% and 5% levels.
(5 marks)

ii.   Do you foresee any difficulties of multicollinearity as a consequence of the definitions of D and G ?   **(2 marks)**

iii.   Discuss possible improvements in the above regression model for share prices.
(3 marks)

(b)   What is meant by auto-correlation in a multiple regression model.? What causes it and what are its effects? Explain how you would identify auto-correlation and how you would test for it.
(10 marks)

END OF PAPER