Examination Papers

Examiners' Reports

UNIVERSITY OF LONDON 18 58 EXTERNAL PROGRAMME

Examination papers and
Examiners' reports

Management mathematics

Economics, Management, Finance and the Social Sciences

2001, 2002, 2003

2790**076**

Examiner's report 2003

Zone A

General remarks
This year's paper was quite standard in its choice of topics and questions. The paper proved to be of a suitable length and it was good to see that the full syllabus seemed to have been prepared by the candidates. Please remember that this paper cannot be taken if either (12) or (D21) or (05b) has been taken and passed.

Question 1

a) i) The initial requirement is to produce a pair of general Venn diagrams of three overlapping sets M, P and S.

Clearly the information that n(M) = 67 must be saying nothing about P or S and $n(M \cap P)=51$ must be saying nothing about S. Similarly, for n(P), n(S), $n(M \cap S)$ and $n(S \cap P)$.

A common omission was to ignore the order of the 'outer' subset, i.e.$M^c \cap S^c \cap P^c$. Setting $n(M \cap P \cap S) = X$ in the 'Workforce', Venn diagram you should get
$n(S^c \cap M \cap P) = 51 – X$, $n(S \cap M^c \cap P) = 3 – X$, $n(S \cap M \cap P^c) = 7–X$,
$n(S^c \cap M^c \cap P) = 16 + X$, $n(S^c \cap M \cap P^c) = 9 + X$, $n(S \cap M^c \cap P^c) = 2 + X$ and finally $n(S^c \cap M^c \cap P^c) = 12 – X$.
Similarly, in the '% of total Salary' Venn diagram, and setting $n(S \cap F \cap P) = Y$, you should get
$n(S^c \cap M \cap P) = 56 – Y$, $n(S \cap M^c \cap P) = 4 – Y$, $n(S \cap M \cap P^c) = 5 –Y$,
$n(S^c \cap M^c \cap P) = 27 + Y$, $n(S^c \cap M \cap P^c) = -1 + Y$, $n(S \cap M^c \cap P^c) = 1 + Y$ and finally $n(S^c \cap M^c \cap P^c) = 5 – Y$.

ii) Note the importance of the phrase "Assuming that each subset of the above Venn diagrams has **positive** order…" and tended to assume non-negativity instead. As a consequence, one might imagine that the maximum value of X is 3 and the minimum value of Y is 1. This answer combination earned only two of the four marks. The required answers were maximum $X = 2$ and minimum $Y = 2$ (or, in this case, perhaps close to, but strictly greater than, 1).

iii) Do not be tempted to guess at the answer for this part without doing the appropriate calculation(s). Also recall the reference to "the eight subsets" created in a) i).

The expected (and correct) answer was that the highest salary per person was in subset $S \cap M^c \cap P$.

b) i) First of all, one needs to determine the salary per person for each year. This is done by taking the total salary bill and dividing by the number of workers. It is important to maintain precision here – do not be tempted to round to the nearest 1,000, for example. The next step is to form a fixed base index (1997=100). This is not requiring the use of Paasche or Laspeyres indices, but merely a comparison (ratio) of salary per person in one year with that in 1997. Thus, for example, the salary per person in 1999 is £1.3 million divided by 55 workers = £23,636, and in 1997 it was £1.2 million divided by 50 = £24,000. Therefore, the index value for 1997 is £23,636 divided by £24,000 = 94.5 (remembering to multiply by 100 to make the value into a standard type of index value).

b) ii) Finally, one needs to make comments about the usefulness or otherwise of the index created. The comments that were hoped for should have included reference to the fact that the workers may change in terms of their experience, skills, age, etc.; inflation should be taken into account; are the workers more efficient? Only one of three available marks was awarded for a narrow view and comment concentrating on the type of index formed.

Question 2

For this sort of standard type differential equation, the main requirement is care in doing all the steps in order. Part (iii) is a more innovative question that requires a wider knowledge of how to use differential equations.

i) It seems appropriate that this report should merely reproduce the key stages of the solution procedure:

Setting $q_D = q_S$ produces the second order differential equation

$$\frac{d^2 p}{dt^2} - 7\frac{dp}{dt} - 60p = -60$$

The auxiliary equation is m^2 - 7m - 60 = 0 which gives two real solutions of 12 or -5 and, hence, a solution of the form $p = Ae^{12t} + Be^{-5t}$.

For a particular solution, we try $p = k$ (there is no need for anything more complicated for this question) and find that $k = 1$.

Combining the two solutions and using the initial conditions to solve for the constants, we find that $A = 3$ and B = 1.

Hence, the required solution is $p = 3e^{12t} + e^{-5t} + 1$.

ii) There **is** no 'long-run' equilibrium since p (and q) tends to infinity as t gets increasingly large. p commences at 5 when $t = 0$ and grows exponentially as t tends to infinity. Remember the simple requirement of labelling the axes of your sketch graph – we know it is fairly obvious what you are plotting, but you will lose credit if the axes are unlabelled.

iii) Do not misjudge the required scope of the comments. For example, the question is not about how differential equations could be used in stock control, etc. The examiners do not regard differential equations (as a general area) as a model. The 'above model' referred to in the question is the specific demand and supply relationship given. Hence, candidates were expected to think of the applicability of such a model and to consider how and why a company might employ it. Good answers would include some reference to the long-run instability. One can also point out that this model might have a limited 'lifetime' since situations (as yet unknown) will cause parameters to change. Hence, the predictive ability of the model may well be time and event (economic) dependent.

Question 3

a) i) Remember to include directional arrows on the arcs of the network and don't include arcs which didn't exist (i.e. there is no flow along them), for example between *A* and *E*.

ii) This is a slightly different question than asked on any previous examination paper for this unit. However, that is not to say it is either unfair or difficult. The correct answer is to determine the transpose of *T*, say *T*', and then to determine *T* + *T*'. As well as being correct, it is a far less time consuming problem than embarking upon the erroneous idea of calculating T^2, for example.

b) i) This question was pleasingly well done. At last, it would appear that more students are making use of the subject guide. A good solution should include a separate analysis for the case of $p = q$ and $p \neq q$, where *p* and *q* are the probabilities of players *A* and *B* winning on each single play of the game.

ii) The important thing here is to realise that the formula derived in (i) is typically based on a gamble of 1 monetary unit per game. Hence, we have to remember that the monetary unit of this specific question is $10. Therefore, using the common notation, $j = 4$, $a = 7$, $p = 0.6$, $q = 0.4$. The required probability of *A* being ruined is 0.148.

Question 4

a) Straight from the subject guide (Chapter 13) – and well remembered by a pleasing number of students! Several others made up a fascinating set of lists of five stages which often enlivened the marking experience. Full marks are obtained only if an explanation (a short one will do) for each of the five stages (Formulation, Estimation, Validation, Forecasting and Implementation) is given.

b) i) The important feature is that this is a 5-point moving average and hence, for example, the smoothed value (forecast) for 1995 is (1990 + 1991 + 1992 + 1993 + 1994)/5= 8.48. In a similar fashion, one can produce a smoothed value for the years 1996 through to 2003.

ii) The usual problems with exponential smoothing are: getting the appropriate starting point; making certain in this example that you use 0.3 rather than (effectively) using 0.7 as the smoothing constant; and making certain that you do not find yourself using the actual revenue for a year when forecasting the revenue for that same year! In this example, it is appropriate to start off with 'forecasts of 8.3 for 1991 and thereafter make the forecast for year *t* equal to 0.3 (revenue in year *t* -1) + 0.7(prediction for year *t* - 1). Thus, the forecast for 1992 becomes 0.3(8.2) + 0.7(8.3) = 8.27 and so forth. Continuing in this mode (and making certain to retain sufficient accuracy), one gets a forecast for 2002 of 13.52. Once again, we can then produce a forecast for 2004 as 13.84.

iii) This part requires a short discussion of the differences between the two methods and their forecasting ability. Many candidates quite rightly mentioned the ability of these methods to react to changes in the revenue figures, although both methods lag behind the actual changes that occur. A reference to either Root Mean Square Error or Mean Absolute Deviation was required to earn the final mark of the three.

iv) Suggest some other variables such as price, advertising, competitors, actions, etc. Perhaps suggest the use of lagged variables and even go so far as to produce an equation to use in the forecasting model.

Question 5

Not a particularly elegant question and rather lacking in an obvious application to management. Nonetheless, it proved to be quite popular and reasonably well done. Lack of accuracy seemed to be as much a problem as any lack of basic knowledge or recognition of what was required. In particular, care needs to be taken with signs and in taking the expansion sufficiently far.

a) i) For this part, the expected answer was for expanding *f(x)* about *a*

$$f(x) = f(a) + \frac{(x-a)}{1!}f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a)...$$

although Maclaurin's theorem obtained by setting $a = 0$ was also allowed.

Hence we find that

$$e^x = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!}...$$

and $$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$$

b) Using the above and by inserting an expansion for 1 - cos *x* in as the *x* for the expansion of e^x we find that $e^{1-\cos x} = 1 + \frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{720}.....$
Integrating this expansion term by term and inserting $x = \pi/6$ will give a value for the integral of 0.5483.

c) i) Multiplying top and bottom by the complex conjugate (i.e. 3 + 2i), we get a real part of 6/13 and an imaginary part of 17/13. The Argand diagram for this complex number should have the horizontal axis as the real part and the vertical axis as the imaginary part (and not vice versa). Both axes should be labelled and a line is required between the origin and the coordinate point (6/13, 17/13). The line should not go beyond the point.

ii) The key here is to recognise that $\frac{1}{2}(1-i\sqrt{3}) = e^{\frac{5i\pi}{3}}$.

Taking the natural logarithm of this merely gives the result that the real part of the number is zero and the imaginary part is the exponent of the exponential, i.e. $\frac{5\pi}{3}$.

Question 6

a) i) The logistic growth type curve that requires sketch graphing is a common type for explaining market penetration or sales volume from the launch of a new product until it reaches a stable equilibrium amount. The curve should show that that at time $t = 0$, $y = 1818.2$, and when t tends to infinity, y is asymptotic to 20,000.The curve is monotonically increasing as t increases.

ii) Take care with the coefficients in the formula – it is easy to get the 2s and 4s in the wrong place. With care, you should get the answer 12,524.01.

b) Take care not to confuse the two surpluses! Setting the supply and demand price equation equal (note that this means setting the *P*s equal, not setting $P = 5P$!), one finds that the equilibrium quantity should be 6 and the equilibrium price 8. The Consumers' Surplus works out to be 42 and the Producers' Surplus is 28.8. A diagram is not required but sometimes helps to avoid mistakes.

Question 7

Perhaps the notation or length of question is discouraging. However, it is quite straightforward although a little 'messy' in terms of the arithmetic.

i) If we let the equilibrium proportions be x^*, y^* and z^* then

$$\begin{pmatrix} x^* & y^* & z^* \end{pmatrix} = \begin{pmatrix} x^* & y^* & z^* \end{pmatrix} \begin{pmatrix} 0.2 & 0.9 & 0 \\ 0.5 & 0 & 0.8 \\ 0.3 & 0.1 & 0.2 \end{pmatrix}$$

and $x^* + y^* + z^* = 1$. In either case, the process of solving these equations is simple and, for example, the equilibrium proportions are $x^* = y^* = z^* = 1/3$, i.e. one third of the total contracts each. In other words, if x^*, y^* and z^* become (or, more likely, start off) the same, then we have equilibrium.

ii) Using the given matrix equation we should have that

$$(x(1) \quad y(1) \quad z(1)) = (x(0) \quad y(0) \quad z(0)).P =$$

$$(112 \quad 80 \quad 56)\begin{pmatrix} 0.2 & 0.9 & 0 \\ 0.5 & 0 & 0.8 \\ 0.3 & 0.1 & 0.2 \end{pmatrix} = (79.2 \quad 106.4 \quad 75.2)$$

In a similar manner, we can use these values to create the numbers at the end of the second month. One should get (91.6 78.8 100.16).

iii) Note that this question specifically asks for P to be inverted in the process of determining *x(t), y(t)* and *z(t)*. It is not sufficient to go as far as an echelon matrix, although this did receive some credit. When inverting the matrix P by row operations, the arithmetic does get rather cumbersome but, ultimately, the answers for *x(t), y(t)* and *z(t)* come out nicely to be 15, 5 and 25 respectively. Inversion of P using adjoints is also allowed, of course.

Question 8

Cluster analysis questions always seem to be popular with students. However, one must always exercise great care: it is easy to make fundamental errors which are heavily penalised.

a) i) One simply needs to calculate the 'distance' between the new observation and each of the existing clusters in turn. Thus, for example, the distance between the new observation and cluster 1 is given by

$$\sqrt{(3-2)^2 + (3.5-1)^2 + (4.4-3)^2} = 3.03\,.$$

Similarly, the distance to cluster 2 is 6.02 and to cluster 3 is 2.21. We would therefore allocate the new observation to cluster 3, since the distance to it is the least of the three.

ii) A repeat of the exercise in (i), except we have a new distance measure. The distances to clusters 1, 2 and 3 now turn out to be 4.9, 9.5 and 3.3 respectively and, hence, one would put the new observation in with cluster 3.

b) Candidates need to exercise greater care in these types of questions and make certain they are clustering in an appropriate method. Perhaps greater care in reading the question is required by these candidates.

This particular clustering question requires cities to be grouped together and the table in the question gives distances between these cities. It would therefore not be sensible to group cities that are far apart. Hence, in each step of the procedure, irrespective of whether single or complete linkage is being used, the best pair of cities (or clusters) to join will be based on the minimum measure of distance between them. The difference between the single and complete linkage methods lies in the way in which the distance measure between clusters is determined. The single linkage takes the distance between two clusters as the **minimum** distance between a pair of cities, one chosen from each cluster. The complete linkage method takes the distance between two clusters as the **maximum** distance between any pair of cities, one chosen from each cluster.

More students need to recognise that there are two aspects of hierarchical clustering, namely (i) the choice of what to cluster next (this is based upon the logic of the question and whether we are concerned with closeness, similarity, distance, etc.), and (ii) the choice of how to measure the distance between clusters (this is where the single and complete clustering terminology applies).

Once we have the two clustering processes worked out for the question, one needs to make some reference back to the fact that they give different answers (in this case). Most students automatically produced a dendogram for each clustering process and this was accepted as a clear indication that the two methods are producing different answers.

Concluding remarks
There is no intention to change the nature of the paper (either in terms of format or style of questions) for next year. However, the examiners once again urge candidates to cover the whole syllabus.