

International Institute for
Technology and Management



Tutoring Sheet # 9

Answers

Unit 05a : Mathematics 1

Evaluate the following integrals:

$$1. \int (3x^2 + 2x - 5)dx = 3\frac{x^3}{3} + 2\frac{x^2}{2} - 5x + C = x^3 + x^2 - 5x + C$$

$$2. \int x\sqrt{x}dx = \int x^1 \times x^{\frac{1}{2}}dx = \int x^{\frac{3}{2}}dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C = \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2x^{\frac{5}{2}}}{5} + C$$

$$3. \int_0^4 \frac{dx}{\sqrt{x}} = \int_0^4 \frac{dx}{x^{\frac{1}{2}}} = \int_0^4 x^{\frac{-1}{2}}dx = \frac{x^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2\sqrt{x} = F(4) - F(0)$$
$$= 2\sqrt{4} - 2\sqrt{0} = 2(2) - 0 = 4$$

$$4. \int_0^2 x(x^4 + 2)dx = \int_0^2 (x^5 + 2x)dx = \frac{x^6}{6} + 2\frac{x^2}{2} = \frac{x^6}{6} + x^2 = F(2) - F(0)$$
$$= \left(\frac{2^6}{6} + 2^2\right) - \left(\frac{0^6}{6} + 0^2\right) = \frac{64}{6} - 0 = \frac{32}{3}$$

$$5. \int \frac{dx}{x^2} = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C = \frac{-1}{x} + C$$

$$6. \int \frac{(x+1)dx}{x^2} = \int (x+1)x^{-2} dx = \int (x^{-1} + x^{-2}) dx = \ln|x| + \frac{x^{-1}}{-1} + C \\ = \ln|x| - \frac{1}{x} + C$$

$$7. \int \frac{x^3 - 3x^2 + 6x}{x} dx = \int (x^3 - 3x^2 + 6x)x^{-1} dx = \int (x^2 - 3x^1 + 6x^0) dx \\ = \frac{x^3}{3} - \frac{3x^2}{2} + 6x + C$$

$$8. \int (x+2)^9 dx \quad ; \text{ Let } u = x + 2 \Rightarrow du = dx \\ = \int u^9 du = \frac{u^{10}}{10} + C = \frac{(x+2)^{10}}{10} + C$$

$$9. \int_1^2 2x(x^2 + 3)^4 dx \quad ; \text{ Let } u = x^2 + 3 \Rightarrow du = 2x dx \\ = \int_1^2 u^4 du = \frac{u^5}{5} = \frac{(x^2 + 3)^5}{5} = F(2) - F(1) \\ = \frac{(2^2 + 3)^5}{5} - \frac{(1^2 + 3)^5}{5} = \frac{7^5}{5} - \frac{4^5}{5} = \frac{7^5 - 4^5}{5}$$

$$10. \int x(3x+5)^7 dx \quad ; \text{ Let } u = 3x + 5 \Rightarrow du = 3 dx \Rightarrow dx = \frac{du}{3} \\ \text{and } x = \frac{u-5}{3} \quad ; \text{ Therefore } \int x(3x+5)^7 dx = \int \frac{u-5}{3} \times u^7 \times \frac{du}{3} \\ = \frac{1}{9} \int (u^8 - 5u^7) du = \frac{1}{9} \left(\frac{u^9}{9} - \frac{5u^8}{8} \right) + C = \frac{u^9}{81} - \frac{5u^8}{27} + C \\ = \frac{(3x+5)^9}{81} - \frac{5(3x+5)^8}{27} + C$$

$$11. \int x (2x^2 + 7)^8 dx$$

$$\text{Let } u = 2x^2 + 7 \Rightarrow du = 4x dx \Rightarrow x dx = \frac{du}{4}$$

$$= \int u^8 \frac{du}{4} = \frac{1}{4} \frac{u^9}{9} + C = \frac{u^9}{36} = \frac{(2x^2 + 7)^9}{36} + C$$

$$12. \int x\sqrt{x-1} dx$$

$$\text{Let } u = \sqrt{x-1} \Rightarrow u^2 = x - 1 \Rightarrow x = u^2 + 1 \\ dx = 2u du$$

$$= \int (u^2 + 1)(u)(2u)du = \int (u^2 + 1)(2u^2)du = 2 \int (u^2 + 1)(u^2)du$$

$$= 2 \int (u^4 + u^2)du = 2 \frac{u^5}{5} + 2 \frac{u^3}{3} + C$$

$$= \frac{2(\sqrt{x-1})^5}{5} + \frac{2(\sqrt{x-1})^3}{3} + C$$