International Institute for
Technology and Management

Tutoring Sheet # 20

Unit 05a : Mathematics 1

SOLUTION

1. Express the following system of equations in a matrix form,and Solve it using a matrix method:

a.) x + y + 3z = 6
2x + y + z = 1
-5x -2y + 2z = 7

b.) x − y + z = 2
3x + y − 2z = 0
5x − 2y − z = 1

c.) 4x + y − 2z = 4
2x + 3y − 2z = 4
2x + 5y + 2z = 8

d.) **x+2y+z= 1**
x + y = 1
3x+4y+z= 3

e.) **x+2y+z= 1**
x + y = 1
3x+4y+z= 2

Please Note Parts d,e

a.) $$\begin{bmatrix} 1 & 1 & 3 & 6 \\ 2 & 1 & 1 & 1 \\ -5 & -2 & 2 & 7 \end{bmatrix} \xrightarrow{-2I_1+I_2} \begin{bmatrix} 1 & 1 & 3 & 6 \\ 0 & -1 & -5 & -11 \\ -5 & -2 & 2 & 7 \end{bmatrix} \xrightarrow{5I_1+I_3}$$

$$\begin{bmatrix} 1 & 1 & 3 & 6 \\ 0 & -1 & -5 & -11 \\ 0 & 3 & 17 & 37 \end{bmatrix} \xrightarrow{3I_2+I_3} \begin{bmatrix} 1 & 1 & 3 & 6 \\ 0 & -1 & -5 & -11 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$

$\therefore$ 2z = 4 $\Rightarrow$ z = 2
-y − 5z = - 11 $\Rightarrow$ -y − 5(2) = - 11 $\Rightarrow$ y = 1
x +y +3 z = 6 $\Rightarrow$ x +1+3(2) = 6 $\Rightarrow$ x = -1

b.) $$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 3 & 1 & -2 & 0 \\ 5 & -2 & -1 & 1 \end{bmatrix} \xrightarrow{-3I_1+I_2} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & -2 & -5 & -6 \\ 5 & -2 & -1 & 1 \end{bmatrix} \xrightarrow{-5I_1+I_3}$$

$$\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & -2 & -5 & -6 \\ 0 & 3 & -6 & 9 \end{bmatrix} \xrightarrow{3I_2+2I_3} \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & -2 & -5 & -6 \\ 0 & 0 & -27 & 0 \end{bmatrix}$$

$\therefore$ -27z = 0 $\Rightarrow$ z = 0
-2y − 5z = - 6 $\Rightarrow$ y = 1
x - y + z = 2 $\Rightarrow$ x = 3

c.) $\begin{bmatrix} 4 & 1 & -2 & 4 \\ 2 & 3 & -2 & 4 \\ 2 & 5 & 2 & 8 \end{bmatrix} \xrightarrow{-I_1+2I_2} \begin{bmatrix} 4 & 1 & -2 & 4 \\ 0 & 2 & 0 & 0 \\ 2 & 5 & 2 & 8 \end{bmatrix} \xrightarrow{-I_1+2I_3}$

$$\begin{bmatrix} 4 & 1 & -2 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 4 & -2 & 4 \end{bmatrix} \xrightarrow{-2I_2+I_3} \begin{bmatrix} 4 & 1 & -2 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -2 & 4 \end{bmatrix}$$

$\therefore$ -2z = 4 $\Rightarrow$ z = -2
2y = 0 $\Rightarrow$ y = 0
4x +y-2z = 4 $\Rightarrow$ x = 0

d.) $\begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 3 & 4 & 1 & 3 \end{bmatrix} \xrightarrow{-I_1+I_2} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 3 & 4 & 1 & 3 \end{bmatrix} \xrightarrow{-3I_1+I_3}$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & -2 & -2 & 0 \end{bmatrix} \xrightarrow{-2I_2+I_3} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ 0x + 0y + 0z = 0 ; any (x,y,z) satisfies this equation
Let z = s
-y –z = 0 $\Rightarrow$ -y –s = 0 $\Rightarrow$ y = -s
x +2y+z = 1 $\Rightarrow$ x +2(-s) + s= 0 $\Rightarrow$ x = s+1

e.) $\begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 3 & 4 & 1 & 2 \end{bmatrix} \xrightarrow{-I_1+I_2} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 3 & 4 & 1 & 2 \end{bmatrix} \xrightarrow{-3I_1+I_3}$

$$\begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & -2 & -2 & -1 \end{bmatrix} \xrightarrow{-2I_2+I_3} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$\therefore$ 0x + 0y + 0z = -1; NO (x,y,z) satisfies this equation
Because we'll end up with 0 = -1
The system has no solution.

2. The supply function for a commodity takes the form : $q^S(p) = ap^2 + bp + c$,when p = 1 the quantity supplied is 5; when p = 2 the quantity supplied is 12 ;when p = 3 the quantity supplied is 23.Find a,b and c using a matrix method.

$p = 1$, $q = 5 \Rightarrow a(1)^2 + b(1) + c = 5 \Rightarrow$ **$a+b+c = 5$**

$p = 2$, $q = 12 \Rightarrow a(2)^2 + b(2) + c = 12 \Rightarrow$ **$4a+2b+c = 12$**

$p = 3$, $q = 23 \Rightarrow a(3)^2 + b(3) + c = 23 \Rightarrow$ **$9a+3b+c = 23$**

$$\begin{bmatrix} 1 & 1 & 1 & 5 \\ 4 & 2 & 1 & 12 \\ 9 & 3 & 1 & 23 \end{bmatrix} \xrightarrow{-4I_1+I_2} \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & -2 & -3 & -8 \\ 9 & 3 & 1 & 23 \end{bmatrix} \xrightarrow{-9I_1+I_3}$$

$$\begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -22 \end{bmatrix} \xrightarrow{-3I_2+I_3} \begin{bmatrix} 1 & 1 & 1 & 5 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$\therefore c = 2$

$-2b-3c = -8 \Rightarrow b = 1$

$a + b + c = 5 \Rightarrow a = 2$

3. Three goods are sold in the same market. If their prices are p_1 , p_2 , p_3 ,then the demanded quantities q_1^D, q_2^D, q_3^D and the supplied quantities q_1^S, q_2^S, q_3^S are given by the equations :

$q_1^D = 45 - 2p_1 + 2p_2 - 2p_3$; $q_1^S = 2p_1 - 5$

$q_2^D = 16 + 2p_1 - p_2 + 2p_3$; $q_2^S = 2p_2 - 4$

$q_3^D = 30 - p_1 + 2p_2 - p_3$; $q_3^S = p_3 - 5$

The equilibrium prices are the non-negative numbers p_1^*, p_2^* , p_3^* with the property that when the prices are $p_1 = p_1^*$, $p_2 = p_2^*$, $p_3 = p_3^*$ then the supply and the demand of each quantity are equal.Using a matrix method find p_1^*, p_2^* , p_3^*

$q_1^D = q_1^S \Rightarrow 45 - 2p_1 + 2p_2 - 2p_3 = 2p_1 - 5$

$\Rightarrow$ **$-4p_1 + 2p_2 - 2p_3 = -50$**

$q_2^D = q_2^S \Rightarrow 16 + 2p_1 - p_2 + 2p_3 = 2p_2 - 4$

$\Rightarrow$ **$2p_1 - 3p_2 + 2p_3 = -20$**

$q_3^D = q_3^S \Rightarrow 30 - p_1 + 2p_2 - p_3 = p_3 - 5$

$\Rightarrow$ **$-p_1 + 2p_2 - 2p_3 = -35$**

Using matrix method: $p_1 = 5$, $p_2 = 60$, $p_3 = 75$