International Institute for
Technology and Management

Tutoring Sheet # 19

Unit 05a : Mathematics 1

1. An arithmetic progression has fifth term equal to 4 ,and the sum of its first 13 terms is 65.Find the first term and the common difference.

a_5 = a + 4d = 4

S_{13} = (13/2)[2a + (13-1)d] = 65 ⇒13a + 78d = 65

Solving the above two equations simultaneously for a and d :

a = 2 , d = ½

2. Find an arithmetic series(first term and common difference) where the fourth term is 5 and the sum of the third and the eighth terms is 1.Then find the 15^{th} term.

a_4 = a +3d = 5

$a_3 + a_8$ = a + 2d + a + 7d = 1 ⇒ 2a + 9d = 1

Solving the above two equations simultaneously for a and d :

a = 14 , d = -3

3. Find three consecutive terms of a geometric sequence such that their product is 64 and their sum is 21.

[Hint: assume the terms : a/r , a , ar)

(a/r)(a)(ar) = 64 ⇒ a^3 = 64 ⇒ a = 4

a/r + a + ar = 21 ⇒ 4/r + 4 + 4r = 21

⇒4 +4r + $4r^2$ = 21r ⇒ $4r^2$ -17r + 4 = 0

⇒ r = 4 or r = ¼

4. In the geometric sequence : 81,27,9, …. Which term is 1/243

a = 81 , r = 27/81 = 1/3

1/243 = ar^{n-1} = $81(1/3)^{n-1}$ ⇒ $(1/3)^{n-1}$ = 1/(243)(81)

$1/3^{n-1}$ = 1/(243)(81) ⇒ $1/3^{n-1}$ = $1/3^5 . 3^4$ = $1/3^9$ ⇒ n - 1 = 9

⇒ n = 10

5. Find a geometric sequence where the third term exceeds the second by 6 and the fourth term exceeds the third by 4.
$a_3 = a_2 + 6 \Rightarrow ar^2 = ar + 6 \Rightarrow a = 6/(r^2 - r)$
$a_4 = a_3 + 4 \Rightarrow ar^3 = ar^2 + 4 \Rightarrow a = 4/(r^3 - r^2)$
$6/(r^2 - r) = 4/(r^3 - r^2) \Rightarrow 6r^3 - 6r^2 = 4r^2 - 4r \Rightarrow 3r^3 - 5r^2 + 2r = 0$
$\Rightarrow r(3r^2 - 5r + 2) = 0 \Rightarrow r = 0$ trivial or $3r^2 - 5r + 2 = 0$
$\Rightarrow r = 1$ or $r = 2/3$

6. A geometric progression has second term equal to 2 and a sum to infinity of 9.Show that there are two possible values of the common ratio and find these.
$a_2 = 2 \Rightarrow ar = 2 \Rightarrow a = 2/r$
$\frac{a}{1-r} = 9 \Rightarrow 9 - 9r = a \Rightarrow 9 - 9r = 2/r \Rightarrow -9r^2 + 9r - 2 = 0$
$\Rightarrow r = 1/3$ or $r = 2/3$

7. An arithmetic progression has first term equal 3 and the sixth term is double the third .Find the sum of the first 9 terms.
$a = 3$; $a_6 = 2a_3 \Rightarrow a + 5d = 2(a + 2d) \Rightarrow a = d$
$\Rightarrow 3 = d$
$S = (9/2)[2a + (n-1)d] = (9/2)[2(3) + 8(3)] = 135$

8. The sum of first n terms of an arithmetic progression is :
$S_n = n^2 - 3n$ .Find the fourth term and the n^{th} term.
$S_1 = a_1$
$S_2 = a_1 + a_2 \Rightarrow a_2 = (a_1 + a_2) - a_1 = \mathbf{S_2 - S_1}$
$S_3 = a_1 + a_2 + a_3 \Rightarrow a_3 = (a_1 + a_2 + a_3) - (a_1 + a_2) = \mathbf{S_3 - S_2}$
Hence $a_n = S_n - S_{n-1} = n^2 - 3n - (n-1)^2 + 3(n-1) = 2n - 4$

9. How many terms are needed of the arithmetic progression 1,3,5,..... to get a sum of 1521.
$S = (n/2)[2a + (n-1)d] = 1521$
$\Rightarrow (n/2)[2 + (n-1)(2)] = 1521 \Rightarrow n^2 = 1521 \Rightarrow n = 39$

10. Find the sum of the first 21 terms of the arithmetic progression:
ln 10 , ln 20 , ln 40 ,………
$a = \ln 10$; $d = \ln 20 - \ln 10 = \ln(20/10) = \ln 2$
$S = (21/2)[2(\ln 10) + (21-1)(\ln 2)] = 21 \ln 10 + 210 \ln 2$

11. An amount of $ 1000 is invested and attracts interest at a rate equivalent to 10% per annum.Find expressions for the total after one year if the interest is compounded :

a.) Annually b.)Quarterly c.)monthly

d.) Daily (assume the year is not a leap year)

What would be the total after **one year** if the interest 10% is Compounded continuously?

a) $A = 1000(1+ 0.1)^1 = 1100$

b) $A = 1000(1+ 0.1/4)^4 = 1103.81$

c) $A = 1000(1+0.1/12)^{12} = 1121.56$

d) $A = 1000(1+ 0.1/365)^{365} = 5795.68$

Compounded continuously :

$P = 1000$, $r = 0.1$, $t = 1$

$$\mathbf{Pe^{rt} = 1000e^{(0.1)(1)} = 1000e^{0.1}}$$

12. Suppose an amount of A_0 is invested and earns an interest of 10% per annum.An additional investment F is added at the end of each subsequent year:

a.) Use the formula of the sum of a geometric progression to derive a formula for the value of the investment A_5 after 5 years.

b.) Derive a formula for the value of the investment A_n after n years with an interest rate *i*% and the additional investment is F .

c.) An investor puts $ 10000 into an investment account that yields interest 10% per annum.The investor adds an additional $5000 by the end of each year. How much will be there in the account by the end of five years?

Show that if the investor has to wait N years until the balance is at least $80000 ,then

$$N \geq \frac{\ln(13/6)}{\ln(1.1)}$$

Answer: Study Guide p. 127

13. A student uses a bank deposit account in which the interest rate is fixed at 4% per annum,with the interest paid at the end of the year (and based on the balance of the account at that time).At the start of the first year he deposits an amount M.
Each year he withdraws an amount W from the account.
Find an expression(involving n) for the amount of money in the account n years after the initial deposit.

If A_n is the required amount then ,

$A_1 = M(1+0.04) - W = 1.04M - W$

$A_2 = A_1(1 + 0.04) - W = (1.04M - W)(1.04) - W$

$A_2 = (1.04)^2 M - (1.04)W - W$

$A_n = (1.04)^n M - (1.04)^{n-1} W - \ldots\ldots - (1.04)W - W$

$A_n = (1.04)^n M - W[1 + 1.04 + 1.04^2 + \ldots\ldots + 1.04^{n-1}]$

Now : $1 + 1.04 + 1.04^2 + \ldots\ldots + 1.04^{n-1}$ is a geometric

Progression of first term a = 1 and ratio r = 1.04

Its sum : $a \times \frac{r^n - 1}{r - 1} = \frac{(1.04)^n - 1}{1.04 - 1} = \frac{(1.04)^n - 1}{0.04}$

$A_n = (1.04)^n M - W[\frac{(1.04)^n - 1}{0.04}] = (1.04)^n M - 25W[1.04)^n - 1]$