



Tutoring Sheet # 18

Unit 05a : Mathematics 1

Solution

1. a.) Use the Lagrange multiplier method to find the values of x and y That maximize the function $f(x,y) = xy^{2/3}$ subject to the constraint $x + 2y = 100$.

$$L = f - \lambda g = xy^{2/3} - \lambda(x + 2y - 100)$$

$$\frac{\partial L}{\partial x} = y^{2/3} - \lambda = 0 \Rightarrow \lambda = y^{2/3}$$

$$\frac{\partial L}{\partial y} = (2/3)xy^{-1/3} - 2\lambda = 0 \Rightarrow \lambda = (1/3)xy^{-1/3}$$

$$\lambda = \lambda \Rightarrow (1/3)xy^{-1/3} = y^{2/3} \text{ divide both sides by } y^{-1/3} :$$

$$(1/3)x = y^{2/3} / y^{-1/3} \Rightarrow x/3 = y^{2/3} \times y^{1/3} \Rightarrow x/3 = y \Rightarrow x = 3y$$

Substitute this in $x + 2y - 100 = 0$

$$3y + 2y = 100 ; y = 20 \text{ and } x = 3y = 3(20) = 60$$

- b.) Use the Lagrange multiplier method to find the values of x and y That maximize the function $f(x,y) = 3\sqrt{x} + 4\sqrt{y}$ subject to the constraint $x + y = 100$.

$$L = f - \lambda g = 3x^{1/2} + 4y^{1/2} - \lambda(x + y - 100)$$

$$\frac{\partial L}{\partial x} = (3/2)x^{-1/2} - \lambda = 0 \Rightarrow \lambda = (3/2)x^{-1/2}$$

$$\frac{\partial L}{\partial y} = 2y^{-1/2} - \lambda = 0 \Rightarrow \lambda = 2y^{-1/2}$$

$$\lambda = \lambda \Rightarrow (3/2)x^{-1/2} = 2y^{-1/2} \Rightarrow 3x^{-1/2} = 4y^{-1/2} \Rightarrow 4x^{1/2} = 3y^{1/2}$$

$$\Rightarrow 4\sqrt{x} = 3\sqrt{y} \text{ Squaring both sides: } 16x = 9y \Rightarrow x = (9/16)y$$

Substitute in $x + y = 100$

$$(9/16)y + y = 100 \Rightarrow 9y + 16y = 1600 \Rightarrow y = 64$$

$$x = (9/16)(64) = 36$$

c.) Optimize $f(x,y) = 120x - 4x^2 + 2xy - 3y^2 + 96y - 222$
 subject to the constraint $x + 3y = 69$

$$L = f - \lambda g = 120x - 4x^2 + 2xy - 3y^2 + 96y - 222 - \lambda(x + 3y - 69)$$

$$\frac{\partial L}{\partial x} = 120 - 8x + 2y - \lambda = 0 \Rightarrow \lambda = -8x + 2y + 120$$

$$\frac{\partial L}{\partial y} = 2x - 6y + 96 - 3\lambda = 0 \Rightarrow \lambda = (2x - 6y + 96)/3$$

$$\lambda = \lambda \Rightarrow -8x + 2y + 120 = (2x - 6y + 96)/3$$

$$\Rightarrow -26x - 12y + 264 = 0 \Rightarrow -13x - 6y + 132 = 0$$

$$\text{Solving: } -13x + 6y + 132 = 0 ; x + 3y - 69 = 0$$

simultaneously: $x = 18$, $y = 17$

d.) Use the Lagrange multiplier method to find the values of x
 and y That maximize the function $f(x,y) = \sqrt{x} y^2$ subject to the
 constraint $x + y = 100$.

$$L = f - \lambda g = x^{1/2} y^2 - \lambda(x + y - 100)$$

$$\frac{\partial L}{\partial x} = (1/2)x^{-1/2} y^2 - \lambda = 0 \Rightarrow \lambda = (1/2)x^{-1/2} y^2$$

$$\frac{\partial L}{\partial y} = 2x^{1/2} y - \lambda = 0 \Rightarrow \lambda = 2x^{1/2} y$$

$$\lambda = \lambda \Rightarrow (1/2)x^{-1/2} y^2 = 2x^{1/2} y , \text{dividing both sides by } 2x^{1/2} y:$$

$$x^{-1} y = 4 \Rightarrow y = 4x , \text{substitute this in : } x + y = 100$$

$$x + 4x = 100 \Rightarrow x = 20 , y = 4x = 4(20) = 80.$$

2. A firm has weekly production function $q(k,l) = k^{1/4} l^{1/2}$ and the unit
 weekly costs for capital and labour are $v = 20$ and $w = 10$.
 the firm wishes to produce 200 units a week of its good. Find the
 minimum cost of doing so.

The total cost : $vk + wl = 20k + 10l$

Minimize $f = 20k + 10l$ subject to $k^{1/4} l^{1/2} = 200$

$$L = f - \lambda g = 20k + 10l - \lambda(k^{1/4} l^{1/2} - 200)$$

$$\frac{\partial L}{\partial k} = 20 - (1/4)\lambda k^{-3/4} l^{1/2} = 0 \Rightarrow \lambda = 80 k^{3/4} l^{-1/2}$$

$$\frac{\partial L}{\partial l} = 10 - (1/2)\lambda k^{1/4} l^{-1/2} = 0 \Rightarrow \lambda = 20 k^{-1/4} l^{1/2}$$

$$\lambda = \lambda \Rightarrow 8k^{3/4} l^{-1/2} = 2k^{-1/4} l^{1/2} \text{ dividing both sides by } 2k^{-1/4} l^{1/2};$$

$$8k^{3/4} l^{-1/2} / 2k^{-1/4} l^{1/2} = 1 \Rightarrow 4k^{3/4} l^{-1/2} k^{1/4} l^{-1/2}$$

$$4kl^{-1} = 1 \Rightarrow 4k/l = 1 \Rightarrow l=4k \text{ substitute this in : } k^{1/4} l^{1/2}=200$$

$$k^{1/4} (4k)^{1/2} = 200 \quad (\text{NB : } 4^{1/2} = \sqrt{4} = 2)$$

$$2k^{3/4} = 200 \Rightarrow k^{3/4} = 100 \Rightarrow k = (100)^{4/3}$$

$$l = 4k = 4(100)^{4/3}$$

3. Study Guide page 95 : Question 5 .

Answer: Study Guide page **101**.

4. Study Guide page 95 : Question 6 .

Answer: Study Guide page **101**.

5. Study Guide page 95 : Question 7 .

Answer: Study Guide page **102**.

6. Study Guide page 95 : Question 8 .

Answer: Study Guide page **103**.