International Institute for
Technology and Management

Tutoring Sheet # 17

Unit 05a : Mathematics 1

Solution

1. A firm has a monopoly for the manufacture of two goods ,X and Y For which the inverse demand functions are :

$P_x = 6 - x$, $P_y = 16 - 2y$

Where P_x and P_y are respective prices.

The firm's cost function is $C(x,y) = 0.5x^2 + 0.5y^2 + xy$

Determine the output quantities (i.e. x and y) which will maximize the firm's profit, and calculate the maximum profit.

Profit = Total Revenue – Total cost

Total Revenue = $xP_x + yP_y = x(6-x) + y(16-2y) = 6x - x^2 + 16y - 2y^2$

Profit = $6x - x^2 + 16y - 2y^2 - 0.5x^2 - 0.5y^2 - xy$

Profit = $-1.5x^2 + 6x - 2.5y^2 + 16y - xy = f(x,y)$

$f_1 = -3x + 6 - y = 0$ ----------(1)

$f_2 = -5y + 16 - x = 0$ ---------(2)

Solving (1) and (2) simultaneously : x = 1 , y = 3

(1,3) maximizes the profit.

$f_{11} = -3$; $f_{22} = -5$, $f_{12} = -1$; $(f_{11})(f_{22}) - f_{12}^{\ 2} = 14 > 0$;

Since $f_{11} = -3 < 0$,(1,3) indeed maximizes f .

The value of the maximum profit: substitute x = 1 , y = 3 in f:

Max Profit = $-1.5(1)^2 + 6(1) - 2.5(3)^2 + 16(3) - (1)(1) = 29$

2. A monopoly manufactures two commodities, X and Y,the markets for which interact. The demand functions are given by :

$x = 8(P_y - P_x)$, $y = 4(9 + 2P_x - 4P_y)$

How much of each commodity should be manufactured to maximize The profit, given that it costs \$1 to produce one unit of x and \$1.5 to produce one unit of y ?

The cost function = (1)(x) + (1.5)(y) = x + 1.5y

Profit = Total Revenue – Total cost

Total Revenue = $xP_x + yP_y$

$x = 8(P_y - P_x)$, $y = 4(9 + 2P_x - 4P_y)$ rearrange as sim. Eq. in P_x and P_y

$-8P_x + 8P_y = x$,

$8P_x - 16P_y = y - 36$ .Solve for P_x & P_y and Proceed as Problem 1.

3. A firm manufactures two products, X and Y , and sells these in related markets.Suppose that the firm is the only producer of X and Y and that the inverse demand functions for X and Y are

$P_x = 13 - 2x - y$, $P_y = 13 - x - 2y$

Determine the production levels that maximize profit,given that the Cost function is $C(x,y) = x + y$

Profit = Total Revenue – Total cost

Total Revenue = $xP_x + yP_y$

$= x(13-2x-y) + y(13- x -2y)=13x-2x^2+13y-2y^2 -2xy$

Profit = $13x-2x^2+13y-2y^2 -2xy - x - y$

Profit = $-2x^2 +12x -2y^2 +12y - 2xy = f(x,y)$

$f_1 = -4x + 12 - 2y = 0$ ----------(1)

$f_2 = -4y +12 - 2x = 0$ ---------(2)

Solving (1) and (2) simultaneously : $x = 2$, $y = 2$

(2,2) maximizes the profit.

$f_{11} = -4$; $f_{22} =-4$, $f_{12} = -2$;$(f_{11})(f_{22}) - f_{12}^{\ 2} = 12 > 0$;

Since $f_{11} = -4 < 0$,(2,2) indeed maximizes f .

The value of the maximum profit: substitute $x = 1$, $y = 3$ in f:

Max Profit = $-2(2)^2 +12(2) -2(2)^2 +12(2) - 2(2)(2) = 28$

4. A data processing company employs both senior and junior Programmers. A particular large project will cost $C = 2000 + 2x^3 - 12xy + y^2$ dollars where x and y represent the Number of junior and senior programmers used respectively. How many employees of each kind should be assigned to the project in order to minimize its cost. What is this minimum cost?

$C = 2000 + 2x^3 - 12xy + y^2 = f(x,y)$

$f_1 = 6x^2 - 12y = 0$ ----------(1)

$f_2 = -12x +2y = 0 \Rightarrow y = 6x$;substituting this in (1) :

$6x^2 - 12(6x) = 0 \Rightarrow x^2 - 12x = 0 \Rightarrow x(x -12) = 0$; $x =0$ or $x = 12$

$x = 0 \Rightarrow y = 6x =6(0) = 0$;(0,,0) trivial : hiring zero programmers.

$x = 12 \Rightarrow y = 6x \Rightarrow y = 6(12) = 72$; (12 , 72) minimizes the cost.

$f_{11} = 12x$; $f_{22} = 2$, $f_{12} = -12$; with $x = 12 \Rightarrow f_{11}=12x =12(12)=144$

$(f_{11})(f_{22}) - f_{12}^{\ 2} = (144)(2)-(-12)^2 = 144 > 0$;

Since $f_{11} = 144 > 0$,(12,72) indeed minimizes f .

The value of the minimum cost: substitute $x = 12$, $y = 72$ in f:

Min Cost = $C = 2000 + 2(12)^3 - 12(12)(72) + (72)^2 = 272$