International Institute for
Technology and Management

Tutoring Sheet # 15

Unit 05a : Mathematics 1

Solution

1. Find the partial derivatives and the second partial derivatives of the following functions :

a. $f(x,y) = x^2y + xy^3$

$f_1 = 2xy + y^3$, $f_2 = x^2 + 3y^2x$,

$f_{11} = 2y$, $f_{12} = 2x + 3y^2$, $f_{21} = 2x + 3y^2$, $f_{22} = 6yx$

b. $f(x,y) = x^3y - \frac{x}{y} = x^3y - xy^{-1}$

$f_1 = 3x^2y - y^{-1}$, $f_2 = x^3 + y^{-2}x$,

$f_{11} = 6xy$, $f_{12} = 3x^2 + y^{-2}$, $f_{21} = 3x^2 + y^{-2}$, $f_{22} = -2y^{-3}x$

c. $f(x,y) = x + \sqrt{y} = x + y^{1/2}$

$f_1 = 1$, $f_2 = (1/2)y^{-1/2}$,

$f_{11} = 0$, $f_{12} = 0$, $f_{21} = 0$, $f_{22} = (-1/4)y^{-3/2}$

d. $f(x,y) = x^{\frac{3}{4}}y^{\frac{1}{4}}$

$f_1 = (3/4)x^{-1/4}y^{1/4}$, $f_2 = (1/4)\, y^{-3/4}x^{3/4}$,

$f_{11} = (-3/16)x^{-5/4}\, y^{1/4}$, $f_{12} = (3/16)x^{-1/4}y^{-3/4}$, $f_{21} = (3/16)x^{-1/4}y^{-3/4}$

$f_{22} = (-3/16)x^{3/4}\, y^{-7/4}$

e. $f(x,y) = x^2\,(x^2 + y^3)^{\frac{2}{3}} = UV$

$f_1 = 2x(x^2 + y^3)^{\frac{2}{3}} + (2/3)x^2(2x)(x^2 + y^3)^{\frac{-1}{3}} = 2x(x^2 + y^3)^{\frac{2}{3}}$

$+ (4/3)x^3(x^2 + y^3)^{\frac{-1}{3}}$

$f_2 = x^2(3y^2)\,(x^2 + y^3)^{\frac{-1}{3}} = 3x^2y^2(x^2 + y^3)^{\frac{-1}{3}}$

$f_{11} = 2(x^2 + y^3)^{\frac{2}{3}} + (2/3)(2x)\,(x^2 + y^3)^{\frac{-1}{3}} + 3(4/3)x^2(x^2 + y^3)^{\frac{-1}{3}}$

$+ (4/3)x^3(-1/3)(2x)\,(x^2 + y^3)^{\frac{-4}{3}}$

$f_{12} = 2x(3y^2)(x^2 + y^3)^{\frac{-1}{3}} + (4/3)x^3(3y^2)(-1/3)(x^2 + y^3)^{\frac{-4}{3}}$

$f_{21} = 6xy^2(x^2 + y^3)^{\frac{-1}{3}} + 3x^2y^2(-1/3)(2x)(x^2 + y^3)^{\frac{-4}{3}}$,

$f_{22} = 6x^2y(x^2 + y^3)^{\frac{-1}{3}} + 3x^2y^2(-1/3)(3y^2)(x^2 + y^3)^{\frac{-4}{3}}$,

f. $f(x,y) = 5x^{\frac{2}{3}}y^{\frac{1}{4}}$

$f_1 = 5(2/3)x^{-1/3}y^{1/4}$, $f_2 = 5(1/4)\,y^{-3/4}x^{2/3}$,
$f_{11} = (-10/9)x^{-4/3}\,y^{1/4}$, $f_{12} = (5/6)x^{-1/3}y^{-3/4}$, $f_{21} = (5/6)x^{-1/3}y^{-3/4}$
$f_{22} = (-15/4)x^{2/3}\,y^{-7/4}$

2. The function f is given by : $f(x,y) = x^{-y}$ for $x > 0$
(Note that: $x^{-y} = e^{-y\ln x}$) *(LSE 2004)*
$f(x,y) = x^{-y} = e^{-y\ln x}$
Find partial derivatives
$\frac{\partial f}{\partial x} = (-y)(1/x)\, e^{-y\ln x}$; $\frac{\partial f}{\partial y} = (-1)(\ln x)\, e^{-y\ln x}$

3. The function f is given by : $f(x,y) = 2^{x^2 y}$ for $x > 0$ *(LSE 2004)*
Find the partial derivatives $f = a^u \Rightarrow f' = u'(\ln a)a^u$
$\frac{\partial f}{\partial x} = (2xy)(\text{Ln}2)\, 2^{x^2 y} = (2\text{Ln}2)xy\, 2^{x^2 y}$
$\frac{\partial f}{\partial y} = (x^2)(\text{Ln}2)\, 2^{x^2 y} = (\text{Ln}2)x^2\, 2^{x^2 y}$

4. The function f is given by : $f(x,y) = \left(\frac{x-y}{x+y}\right)^n$ where $n > 0$

Find the partial derivatives $\frac{\partial f}{\partial x} = n\left(\frac{x-y}{x+y}\right)^{n-1} \times \frac{2y}{(x+y)^2}$

and $\frac{\partial f}{\partial y} = n\left(\frac{x-y}{x+y}\right)^{n-1} \times \frac{-2x}{(x+y)^2}$ *(LSE 2003)*

5. The function f is given by : f(x,y) = $\dfrac{x^2}{y} + \dfrac{\sqrt{x^4 + y^4}}{x + y}$ *(LSE 2003)*

Find the partial derivatives of f and show that $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y}$ = f(x,y)

f = $x^2y^{-1} + (x^4 + y^4)^{1/2}(x+y)^{-1}$

$$\frac{\partial f}{\partial x} = f_1 = 2xy^{-1} + (1/2)4x^3(x^4 + y^4)^{-1/2}(x+y)^{-1} + (x^4 + y^4)^{1/2}(-1)(x+y)^{-}$$

$$\frac{\partial f}{\partial x} = \frac{2x}{y} + \frac{2x^3}{(x+y)\sqrt{x^4 + y^4}} - \frac{\sqrt{x^4 + y^4}}{(x+y)^2}$$

$$x\frac{\partial f}{\partial x} = \frac{2x^2}{y} + \frac{2x^4}{(x+y)\sqrt{x^4 + y^4}} - \frac{x\sqrt{x^4 + y^4}}{(x+y)^2}$$

$$\frac{\partial f}{\partial y} = f_2 = -x^2y^{-2} + (1/2)4y^3(x^4 + y^4)^{-1/2}(x+y)^{-1} + (x^4 + y^4)^{1/2}(-1)(x+y)^{-2}$$

$$\frac{\partial f}{\partial y} = \frac{-x^2}{y^2} + \frac{2y^3}{(x+y)\sqrt{x^4 + y^4}} - \frac{\sqrt{x^4 + y^4}}{(x+y)^2}$$

$$y\frac{\partial f}{\partial y} = \frac{-x^2}{y} + \frac{2y^4}{(x+y)\sqrt{x^4 + y^4}} - \frac{y\sqrt{x^4 + y^4}}{(x+y)^2}$$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f(x,y)$$

For detailed solution refer to Mathematics-1 Examination Book.