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International Institute for
Technology and Management

Tutoring Sheet #11

Answers

Unit 05a : Mathematics 1

1.Show that : $\frac{t^2+4t+4}{t+3} = t+1+\frac{1}{t+3}$

The right hand side: $t+1+\frac{1}{t+3} = \frac{(t+1)(t+3)}{t+3} = \frac{t^2+4t+4}{t+3}$

Hence find: $\int_1^2 \frac{t^2+4t+4}{t+3}\,dt = \int_1^2 \left(t+1+\frac{1}{t+3}\right) dt$

$= \frac{t^2}{2}+t+\ln(t+3)\Big|_1^2 = \frac{2^2}{2}+2+\ln(2+3)-\left(\frac{1^2}{2}+1+\ln(1+3)\right)$

$= 4 + \ln 5 - \frac{3}{2} - \ln 4 = \frac{5}{2}+\ln 5-\ln 4$

2. Determine $\int \frac{x-3}{x^2-6x+5}\,dx$

Let u = x^2 -6x + 5 ⇒ du = (2x-6)dx=2(x-3)dx

⇒ (x-3)dx = $\frac{du}{2}$:

$\int \frac{x-3}{x^2-6x+5}\,dx = \int \frac{\frac{du}{2}}{u} = \frac{1}{2}\ln u + C = \frac{1}{2}\ln(x^2-6x+5)+C$

3. Evaluate the following integrals:

a. $\int \frac{x}{x^2 - x - 2}$ dx by partial fractions:

$$\frac{x}{x^2 - x - 2} = \frac{a}{x+1} + \frac{b}{x-2}$$

$\Rightarrow$ x =a(x-2) + b(x+1)

Choose x = 2 $\Rightarrow$ 2 = a(0) + b(3) $\Rightarrow$ b = $\frac{2}{3}$

Choose x = -1$\Rightarrow$-1 = a(-3) + b(0) $\Rightarrow$ a = $\frac{-1}{3}$

$$\int \frac{x}{x^2 - x - 2}\,\mathrm{dx} = \int \left(\frac{a}{x+1} + \frac{b}{x-2}\right) \mathbf{dx} = \int \left(\frac{\frac{-1}{3}}{x+1} + \frac{\frac{2}{3}}{x-2}\right)\mathbf{dx}$$

$$= \frac{-1}{3}\ln(x+1) + \frac{2}{3}\ln(x-2) + C$$

b. $\int \frac{dx}{x^2 + 4x + 3}$ Using Integration by partial fractions:

$$\frac{1}{x^2 + 4x + 3} = \frac{a}{x+1} + \frac{b}{x+3}$$

$\Rightarrow$ 1 = a(x+3) + b(x+1)

Choose x = -3 $\Rightarrow$ 1 = a(0) + b(-2) $\Rightarrow$ b = $\frac{-1}{2}$

Choose x = -1 $\Rightarrow$ 1 = a(4) + b(0) $\Rightarrow$ a = $\frac{1}{4}$

$$\int \frac{dx}{x^2 + 4x + 3} = \int \left(\frac{\frac{1}{4}}{x+1} + \frac{\frac{-1}{2}}{x+3} \right) dx = \frac{1}{4}\ln(x+1)\ \frac{-1}{2}\ln(x+3) + C$$

c. $\int x^2 e^x dx$ Using integration by parts:

$u=x^2 \Rightarrow du = 2x\,dx$; $dv = e^x\,dx \Rightarrow v = e^x$

$\int u\,dv = uv - \int v\,du$

$\int x^2 e^x dx = x^2 e^x - \int e^x (2x)\,dx = x^2 e^x - 2\int xe^x\,dx$

Now $\int xe^x\,dx$ by parts again:

$u = x \Rightarrow du = dx$; $dv = e^x\,dx \Rightarrow v = e^x$

$\int xe^x\,dx = xe^x - \int e^x\,dx = xe^x - e^x$

$\int x^2 e^x dx = x^2 e^x - 2(xe^x - e^x) + C = (x^2-2x+2)e^x + C$

d. $\int t \ln t\,dt$ Using integration by parts:

$u = \ln t \Rightarrow du = \frac{1}{t}dt$; $dv = t\,dt \Rightarrow v = \frac{t^2}{2}$

$\int u\,dv = uv - \int v\,du$

$\int t \ln t\,dt = \frac{t^2}{2}\ln t - \int \frac{t^2}{2}\frac{1}{t}\,dt = \frac{t^2}{2}\ln t - \frac{1}{2}\int t\,dt$

$= \frac{t^2}{2}\ln t - \frac{1}{2}\frac{t^2}{2} + C = \frac{t^2}{2}\ln t - \frac{t^2}{4} + C$

e. $\int \frac{2x-1}{x^2-x+3}\,dx$

Note that $2x - 1$ is the derivative of $x^2 - x + 3$

Let $u = x^2 - x + 3 \Rightarrow du = (2x - 1)dx$;substituting:

$\int \frac{du}{u} = \ln u + C = \ln(x^2 - x + 3) + C$

f. $\int \frac{\ln x}{x^2}$ dx

View answer at :

http://www.mathyards.com/vb/showthread.php?&threadid=597

g. $\int x^2 \sqrt{x+3}$ dx

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h. $\int x \sin x$ dx Using integration by parts:

u = x ⇒ du = dx ; dv = sinx dx ⇒ v = $\int \sin x dx$

⇒ v = - cosx

$\int u\, dv = uv - \int v\, du$

$\int x \sin x$ dx = -x cosx - $\int -\cos x\, dx$

= -x cosx + $\int \cos x dx$

= -x cos + sinx + C

END of ANSWERS