

International Institute for
Technology and Management



Tutoring Sheet #11

Answers

Unit 05a : Mathematics 1

1. Show that : $\frac{t^2 + 4t + 4}{t + 3} = t + 1 + \frac{1}{t + 3}$

The right hand side: $t + 1 + \frac{1}{t + 3} = \frac{(t + 1)(t + 3) + 1}{t + 3} = \frac{t^2 + 4t + 4}{t + 3}$

Hence find: $\int_1^2 \frac{t^2 + 4t + 4}{t + 3} dt = \int_1^2 \left(t + 1 + \frac{1}{t + 3} \right) dt$

$$= \left. \frac{t^2}{2} + t + \ln(t + 3) \right|_1^2 = \frac{2^2}{2} + 2 + \ln(2 + 3) - \left(\frac{1^2}{2} + 1 + \ln(1 + 3) \right)$$

$$= 4 + \ln 5 - \frac{3}{2} - \ln 4 = \frac{5}{2} + \ln 5 - \ln 4$$

2. Determine $\int \frac{x - 3}{x^2 - 6x + 5} dx$

Let $u = x^2 - 6x + 5 \Rightarrow du = (2x - 6)dx = 2(x - 3)dx$

$$\Rightarrow (x - 3)dx = \frac{du}{2} :$$

$$\int \frac{x - 3}{x^2 - 6x + 5} dx = \int \frac{\frac{du}{2}}{u} = \frac{1}{2} \ln u + C = \frac{1}{2} \ln(x^2 - 6x + 5) + C$$

3. Evaluate the following integrals:

a. $\int \frac{x}{x^2 - x - 2} dx$ by partial fractions:

$$\frac{x}{x^2 - x - 2} = \frac{a}{x+1} + \frac{b}{x-2}$$

$$\Rightarrow x = a(x-2) + b(x+1)$$

$$\text{Choose } x = 2 \Rightarrow 2 = a(0) + b(3) \Rightarrow b = \frac{2}{3}$$

$$\text{Choose } x = -1 \Rightarrow -1 = a(-3) + b(0) \Rightarrow a = \frac{-1}{3}$$

$$\int \frac{x}{x^2 - x - 2} dx = \int \left(\frac{a}{x+1} + \frac{b}{x-2} \right) dx = \int \left(\frac{-1}{3} \frac{1}{x+1} + \frac{2}{3} \frac{1}{x-2} \right) dx$$

$$= \frac{-1}{3} \ln(x+1) + \frac{2}{3} \ln(x-2) + C$$

b. $\int \frac{dx}{x^2 + 4x + 3}$ Using Integration by partial fractions:

$$\frac{1}{x^2 + 4x + 3} = \frac{a}{x+1} + \frac{b}{x+3}$$

$$\Rightarrow 1 = a(x+3) + b(x+1)$$

$$\text{Choose } x = -3 \Rightarrow 1 = a(0) + b(-2) \Rightarrow b = \frac{-1}{2}$$

$$\text{Choose } x = -1 \Rightarrow 1 = a(4) + b(0) \Rightarrow a = \frac{1}{4}$$

$$\int \frac{dx}{x^2 + 4x + 3} = \int \left(\frac{1}{4} \frac{1}{x+1} + \frac{-1}{2} \frac{1}{x+3} \right) dx = \frac{1}{4} \ln(x+1) - \frac{1}{2} \ln(x+3) + C$$

c. $\int x^2 e^x dx$ Using integration by parts:

$$u=x^2 \Rightarrow du = 2x dx ; dv = e^x dx \Rightarrow v = e^x$$

$$\int u dv = uv - \int v du$$

$$\int x^2 e^x dx = x^2 e^x - \int e^x (2x) dx = x^2 e^x - 2 \int x e^x dx$$

Now $\int x e^x dx$ by parts again:

$$u = x \Rightarrow du = dx ; dv = e^x dx \Rightarrow v = e^x$$

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x$$

$$\int x^2 e^x dx = x^2 e^x - 2(x e^x - e^x) + C = (x^2 - 2x + 2)e^x + C$$

d. $\int t \ln t dt$ Using integration by parts:

$$u = \ln t \Rightarrow du = \frac{1}{t} dt ; dv = t dt \Rightarrow v = \frac{t^2}{2}$$

$$\int u dv = uv - \int v du$$

$$\int t \ln t dt = \frac{t^2}{2} \ln t - \int \frac{t^2}{2} \frac{1}{t} dt = \frac{t^2}{2} \ln t - \frac{1}{2} \int t dt$$

$$= \frac{t^2}{2} \ln t - \frac{1}{2} \frac{t^2}{2} + C = \frac{t^2}{2} \ln t - \frac{t^2}{4} + C$$

e. $\int \frac{2x-1}{x^2-x+3} dx$

Note that $2x - 1$ is the derivative of $x^2 - x + 3$

Let $u = x^2 - x + 3 \Rightarrow du = (2x - 1)dx$; substituting:

$$\int \frac{du}{u} = \ln u + C = \ln(x^2 - x + 3) + C$$

$$f. \int \frac{\ln x}{x^2} dx$$

View answer at :

<http://www.mathyards.com/vb/showthread.php?&threadid=597>

$$g. \int x^2 \sqrt{x+3} dx$$

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h. $\int x \sin x dx$ Using integration by parts:

$$u = x \Rightarrow du = dx \quad ; \quad dv = \sin x dx \Rightarrow v = \int \sin x dx$$

$$\Rightarrow v = -\cos x$$

$$\int u dv = uv - \int v du$$

$$\int x \sin x dx = -x \cos x - \int -\cos x dx$$

$$= -x \cos x + \int \cos x dx$$

$$= -x \cos x + \sin x + C$$

END of ANSWERS