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International Institute for
Technology and Management

Tutoring Sheet #10

Answers

Unit 05a : Mathematics 1

1. $\int (x^3 - 14x^2 + 20x + 3)dx = \frac{x^4}{4} - \frac{14x^3}{3} + \frac{20x^2}{2} + 3x + C$

2. $\int (2 + e^{-x})\, dx$ = 2x – e^{-x} + C

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

3. $\int (\frac{1}{t^2} - \frac{2}{\sqrt{t}})dt = \int (t^{-2} - 2t^{\frac{-1}{2}})dt = \frac{t^{-2+1}}{-2+1} - 2\frac{t^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + C$

$= \frac{t^{-1}}{-1} - 2\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{-1}{t} - 4t^{\frac{1}{2}} + C$

4. $\int_0^T qe^{qt} dt$ = q$\int_0^T e^{qt} dt$ = $q \times \frac{1}{q} e^{qt} \Big]_0^T = e^{qt} \Big]_0^T = e^{qT} - e^0 = e^{qT} - 1$

5. $\int (6x^{-3} + 4x^{-1})dx$ = $6\frac{x^{-3+1}}{-3+1} + 4\ln x + C = \frac{6x^{-2}}{-2} + 4\ln x + C$

= $-3x^{-2}$ + 4lnx + C

$$\int \frac{dx}{x} = \int x^{-1} dx = \ln x + C$$

6. $\int (\frac{3}{x} + 4e^{-5x})dx$ = 3lnx + $\frac{4}{-5}e^{-5x} + C$ =3lnx $-\frac{4}{5}e^{-5x} + C$

7. $\int \frac{dt}{1-2t}$ = $\frac{-1}{2}\ln(1-2t) + C$

$$\int \frac{dx}{b+ax} = \frac{1}{a}\ln(b+ax) + C$$

8. $\int \frac{7}{3-4t}dt$ = $7 \times \frac{1}{-4}\ln(2-4t)$ + C = $\frac{-7}{4}\ln(2-4t)$ + C

9. $\int 8 x e^{3x^2}$ dx   Let u = $x^2 \Rightarrow$ du = 2xdx $\Rightarrow$ xdx = $\frac{du}{2}$

= $\int 8\, e^{3u}\frac{du}{2}$ =4$\int e^{3u}du$ = 4$\times \frac{1}{3}e^{3u} + C$ = $\frac{4}{3}e^{3x^2} + C$

10. $\int \frac{3x^2+2}{x^3+2x-5}dx$

Note that $3x^2 + 2$ is the derivative of $x^3 + 2x - 5$

Let u = $x^3 + 2x - 5 \Rightarrow$ du = $(3x^2 + 2)$dx ;substituting:

$\int \frac{du}{u} = \ln u + C = \ln(x^3 + 2x - 5) + C$

11. $\int \sqrt{1-x}\,dx$ Let u = 1-x $\Rightarrow$ du = -dx$\Rightarrow$dx = -du

$\int \sqrt{u}(-du)$ = - $\int \sqrt{u}du$ $= -\int u^{\frac{1}{2}}du$ = $-\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$

= $\frac{-2}{3}u^{\frac{3}{2}} + C$ = $\frac{-2}{3}(\sqrt{1-x})^{\frac{3}{2}} + C$

12. $\int x^2(x^3+1)^4\,dx$

Let $u = x^3 + 1 \Rightarrow du = 3x^2\,dx \Rightarrow x^2\,dx = \frac{du}{3}$

$$\int u^4 \frac{du}{3} = \frac{1}{3}\int u^4 du = \frac{1}{3}\frac{u^5}{5} + C = \frac{u^5}{15} + C = \frac{(x^3+1)^5}{15} + C$$

13. $\int \frac{e^{2t}}{e^{2t}+1}\,dt$

Let $u = e^{2t} + 1 \Rightarrow du = 2e^{2t}\,dt \Rightarrow e^{2t}\,dt = \frac{du}{2}$

$$\int \frac{\frac{1}{2}du}{u} = \frac{1}{2}\ln u + C = \frac{1}{2}\ln(e^{2t}+1) + C$$

14. $\int_1^e \frac{3}{x}\,dx = 3\ln x\Big|_1^e = 3\ln e - 3\ln 1 = 3(1) - 3(0) = 3-0 = 3$

Ln e = 1 and ln 1 = 0

END of ANSWERS