



November 27th, 2005

Unit: 05a – Mathematics 1

GROUP(C)-VERSION C

This paper is not to be removed from the Examination Halls

Student Name :

Student Number :

Tuesday 27th November 7 : 00 pm – 9 : 00 pm

Candidates should answer **NINE** of the following **ELEVEN** questions: **SEVEN** from section A (60 marks in total) and **TWO** from section B (20 marks each).

Candidates are strongly advised to divide their time accordingly.

Graph paper is provided at the student request.

Calculators **May NOT** be used for this paper.

PLEASE TURN OVER

SECTION A

Answer all **SEVEN** questions from this section (60 marks in total)

1. The supply equation for a good is

$$q = p^2 - 14p - 4$$

and the demand equation is

$$q = -p^2 + 2p + 50$$

where p is the price.

Sketch the supply and the demand functions for $p \geq 0$

Determine the equilibrium price and quantity.

2. Find the value of x that maximises the function:

$$f(x) = (1 + x)e^{-x/4}$$

Verify that it is a maximum.

3. Determine the following integrals

$$\int \frac{dx}{x(2 - \ln x)^3}, \int \frac{\cos x}{\sqrt{1 + \sin x}} dx$$

4. A firm has average variable cost

$$q^2 + 2q + \frac{e^{q^2+1}}{q}$$

and fixed costs of 8. Find the total cost function and the marginal cost function.

5. A firm has marginal cost qe^{q^2} and fixed costs are 5.
Find its total cost function.

6. A monopolist's cost function is given by :

$$q^2 - 1$$

Where q is the quantity produced, the inverse demand function for the good is $p = 32 - 7q$

Determine expressions, in terms of q , for the revenue and The profit and determine the value of q that maximizes the profit. Find the maximum profit.

7. Determine the following integral

$$\int \frac{x+1}{x^2+2x+5} dx, \int \sin^2 x \cos^5 x dx$$

SECTION B

Answer **TWO** questions from this section (20 marks each)

8. (a) A firm has average variable cost

$$2q^2 + 5q + \frac{\ln(q^3 + 2)}{q}$$

and fixed costs of 4. Find the total cost function and the marginal cost function.

- (b) Determine the following integrals

$$\int x^2 \sqrt{2x+1} dx, \int \frac{e^x}{(e^x + 1)^2} dx$$

9. (a) A firm's demand function is

$$p = aq + b \quad (a < 0 ; b > 0)$$

fixed costs are c and variable costs are d per unit.

Show that the profit is maximized when $q = \frac{d-b}{2a}$

- (b) Find the critical points of the function:

$$f(x) = x^4 - 8x^3 - 80x^2 + 15$$

and specify their nature.

10. (a) A firm's marginal revenue function is $MR = 11 - q$

The firm's marginal cost function is

$$MC = q^2 - 3q + 3$$

where q is either the quantity sold or produced.

Find the profit-maximizing level of output and verify that it is a maximum.

- (b) Determine the following integrals

$$\int_1^2 \frac{(\ln x)^3}{x} dx, \int x^2 e^x dx$$

11. A firm faces a total cost function $TC = \frac{1}{2} q^3 - 4q^2 + 6q$

(i) Determine the firm's average cost (AC) and marginal cost (MC) functions.

(ii) Sketch the average cost (AC) and the marginal cost (MC) on the same graph.

(iii) If price is \$ 15, which level of output will a profit maximising firm choose?

END OF PAPER