



November 2008

Unit: 05a – Mathematics 1

# GROUP(B)-VERSION B

This paper is not to be removed from the Examination Halls

Student Name :

Student Number :

TIME ALLOWED: 2 hours

Candidates should answer **NINE** of the following **ELEVEN** questions: **SEVEN** from section A (60 marks in total) and **TWO** from section B (20 marks each).

Candidates are strongly advised to divide their time accordingly.

Graph paper is provided at the student request.

Calculators **May NOT** be used for this paper.

**PLEASE TURN OVER**

## SECTION A

Answer all **SEVEN** questions from this section (60 marks in total)

1. The supply equation for a good is

$$q = p^2 - 4$$

and the demand equation is

$$q = p + 2$$

where  $p$  is the price.

Sketch the supply and the demand functions for  $p \geq 0$

Determine the equilibrium price and quantity.

2. Find the critical points of the following function:

$$f(x) = x^3 e^{-x}$$

and specify their nature.

3. Determine the following integrals

$$\int x\sqrt{1-x} dx \quad , \quad \int \frac{dx}{x(2-\ln x)^3}$$

4. The total cost function of a firm is

$$5000 + 15q + q^2 \sqrt{1+2q}$$

Find the average cost and the marginal cost functions.

5. Find the positive number  $a$  which is such that

$$\int_1^a \left( 1 + \frac{2}{x^2} \right) dx = 2$$

6. The demand function for a company is

$$q = 1000e^{-0.2p}$$

the fixed costs are 100 and the variable costs are 2 per unit. Find the price needed to maximize the profit.

7. Determine the following integrals

$$\int \frac{x \ln(1+x^2)}{1+x^2} dx \quad , \quad \int \frac{\sin x dx}{\cos^4 x}$$

## SECTION B

Answer **TWO** questions from this section (20 marks each)

8. (a) A firm is a monopoly for the good it produces, It has a marginal cost function  $MC = q - 5$  and fixed costs of **300**. The demand equation for its good is given by  $q = 850 - 10p$  where  $p$  is the price. Find expressions in terms of  $q$ , for the total revenue and profit. Determine the value of  $q$  that maximises the profit. Calculate this maximum profit.

- (b) Determine the following integrals

$$\int \frac{x+3}{(x^2+6x+5)^2} dx, \quad \int \frac{dx}{e^x(2+e^{-x})}$$

9. (a) A monopolist's cost function is given by :  $q^2 - 1$   
Where  $q$  is the quantity produced, the inverse demand function for the good is  $p = 32 - 7q$   
Determine expressions, in terms of  $q$ , for the revenue and The profit and determine the value of  $q$  that maximizes the profit. Find the maximum profit.

- (b) Find and classify the stationary points of the function  
 $f(x) = 3x^4 - 4x^3 - 12x^2 + 9$

10. (a) A firm's marginal revenue function is given by:  
 $MR = 3/2$ . The firm's total cost function is given by:  
 $TC = 6 + q + 4q^2 + 6q^3$   
where  $q$  is either the quantity sold or produced.  
Find the quantity that maximises the profit and verify that it is a maximum.

(b) Show that :  $\frac{x^3}{x^2-1} = x + \frac{x}{x^2-1}$

Then find :  $\int \frac{x^3}{x^2-1} dx$

11. A firm faces a total cost function  $TC = 20 + 5q + 5q^2$
- Determine the firm's average cost (AC) and marginal cost (MC) functions.
  - Find the quantity that minimises the Average cost and the value of this minimum. Show indeed it is a minimum.
  - Verify that when  $q = 2$ , the marginal cost MC equals the Average cost.
  - Sketch the graphs of the total cost TC and the marginal cost MC functions on the same system of axes.

**END OF PAPER**