



November 2008

Unit: 05a – Mathematics 1

GROUP(B)-VERSION B

This paper is not to be removed from the Examination Halls

Student Name :

Student Number :

TIME ALLOWED: 2 hours

Candidates should answer **NINE** of the following **ELEVEN** questions: **SEVEN** from section A (60 marks in total) and **TWO** from section B (20 marks each).

Candidates are strongly advised to divide their time accordingly.

Graph paper is provided at the student request.

Calculators **May NOT** be used for this paper.

PLEASE TURN OVER

SECTION A

Answer all **SEVEN** questions from this section (60 marks in total)

1. The supply equation for a good is

$$\mathbf{q = p^2 - 4}$$

and the demand equation is

$$\mathbf{q = p + 2}$$

where p is the price.

Sketch the supply and the demand functions for $p \geq 0$

Determine the equilibrium price and quantity.

2. Find the critical points of the following function:

$$f(x) = x^3 e^{-x}$$

and specify their nature.

3. Determine the following integrals

$$\int x\sqrt{1-x} dx \quad , \quad \int \frac{dx}{x(2-\ln x)^3}$$

4. The total cost function of a firm is

$$\mathbf{5000 + 15q + q^2 \sqrt{1+2q}}$$

Find the average cost and the marginal cost functions.

5. Find the positive number a which is such that

$$\int_1^a \left(1 + \frac{2}{x^2} \right) dx = 2$$

6. The demand function for a company is

$$\mathbf{q = 1000e^{-0.2p}}$$

the fixed costs are 100 and the variable costs are 2 per unit. Find the price needed to maximize the profit.

7. Determine the following integrals

$$\int \frac{x \ln(1+x^2)}{1+x^2} dx \quad , \quad \int \frac{\sin x dx}{\cos^4 x}$$

SECTION B

Answer **TWO** questions from this section (20 marks each)

8. (a) A firm is a monopoly for the good it produces, It has a marginal cost function $MC = q - 5$ and fixed costs of **300**. The demand equation for its good is given by $q = 850 - 10p$ where p is the price. Find expressions in terms of q , for the total revenue and profit. Determine the value of q that maximises the profit. Calculate this maximum profit.

- (b) Determine the following integrals

$$\int \frac{x+3}{(x^2+6x+5)^2} dx, \quad \int \frac{dx}{e^x(2+e^{-x})}$$

9. (a) A monopolist's cost function is given by : $q^2 - 1$
Where q is the quantity produced, the inverse demand function for the good is $p = 32 - 7q$
Determine expressions, in terms of q , for the revenue and The profit and determine the value of q that maximizes the profit. Find the maximum profit.

- (b) Find and classify the stationary points of the function
 $f(x) = 3x^4 - 4x^3 - 12x^2 + 9$

10. (a) A firm's marginal revenue function is given by:
 $MR = 3/2$. The firm's total cost function is given by:
 $TC = 6 + q + 4q^2 + 6q^3$
where q is either the quantity sold or produced.
Find the quantity that maximises the profit and verify that it is a maximum.

(b) Show that : $\frac{x^3}{x^2-1} = x + \frac{x}{x^2-1}$

Then find : $\int \frac{x^3}{x^2-1} dx$

11. A firm faces a total cost function $TC = 20 + 5q + 5q^2$
- Determine the firm's average cost (AC) and marginal cost (MC) functions.
 - Find the quantity that minimises the Average cost and the value of this minimum. Show indeed it is a minimum.
 - Verify that when $q = 2$, the marginal cost MC equals the Average cost.
 - Sketch the graphs of the total cost TC and the marginal cost MC functions on the same system of axes.

END OF PAPER