



November 2008

Unit: 05a – Mathematics 1

GROUP(A)-VERSION A

This paper is not to be removed from the Examination Halls

Student Name :

Student Number :

TIME ALLOWED: 2 hours

Candidates should answer **NINE** of the following **ELEVEN** questions: **SEVEN** from section A (60 marks in total) and **TWO** from section B (20 marks each).

Candidates are strongly advised to divide their time accordingly.

Graph paper is provided at the student request.

Calculators **May NOT** be used for this paper.

PLEASE TURN OVER

SECTION A

Answer all **SEVEN** questions from this section (60 marks in total)

1. The functions $f(x)$ and $g(x)$ are:

$$f(x) = 2x^2 + x - 10, \quad g(x) = 7 - 3x^2 - 4x$$

Sketch the graphs of f and g , and determine the x -coordinates of their points of intersection.

2. Find the maximum value of the function:

$$f(x) = (1+x)e^{\frac{-x}{4}}$$

Show that it is indeed a maximum.

3. Determine the following integrals

$$\int \frac{\cos(\ln x)}{x} dx, \quad \int x^3 \sqrt{4x^2 + 1} dx$$

4. A firm has average variable cost

$$2q^2 + 5q + \frac{\ln(q^3 + 2)}{q}$$

and fixed costs of 4. Find the total cost function and the marginal cost function.

5. The marginal cost is a function of output as follows :

$$MC = 10 - q + q^2$$

Determine the extra cost which is incurred when production is increased from 2 to 4.

6. Find the positive number a which is such that

$$\int_1^a \left(1 + \frac{2}{x^2} \right) dx = 2$$

7. Determine the following integrals

$$\int \frac{2\sqrt{x} + 1}{\sqrt{x}(x + \sqrt{x} - 2)} dx, \quad \int \frac{\cos x dx}{\sin^2 x + 2 \sin x + 1}$$

SECTION B

Answer **TWO** questions from this section (20 marks each)

8. (a) A monopoly has fixed costs of **10** and marginal cost function $3q^2 + 4$. The demand equation for its product is $p + q = 20$. Determine the profit function in terms of q . Determine also the production level that maximises the profit.

- (b) Determine the following integrals

$$\int \sin^2 x \cos^5 x dx \quad , \quad \int \frac{e^x dx}{\sqrt{e^x + 1}}$$

9. (a) A monopoly has fixed costs of **10** and Average variable cost function $q^2 + 4$. The demand equation for its product is $p + q = 20$. Determine the profit function in terms of q . Determine also the production level that maximises the profit.

- (b) Find the critical points of the function and specify their nature:

$$f(x) = x^4 - 8x^3 - 80x^2 + 15$$

10. (a) A firm's marginal revenue function is $MR = 11 - q$. The firm's marginal cost function is $MC = 3q^2 + 36q - 36$ where q is either the quantity sold or produced. Find the value of q which maximises the profit. Determine the maximum profit and verify that it is a maximum.

(b) **Show that** :
$$\frac{2x^2 - 3x + 4}{x - 1} = 2x - 1 + \frac{3}{x - 1}$$

Then find :
$$\int \frac{2x^2 - 3x + 4}{x - 1} dx$$

11. A firm faces a total cost function $TC = 20 + 5q + 5q^2$
- Determine the firm's average cost (AC) and marginal cost (MC) functions.
 - Find the quantity that minimises the Average cost and the value of this minimum. Show indeed it is a minimum.
 - Verify that when $q = 2$, the marginal cost MC equals the Average cost.
 - Sketch the graphs of the total cost TC and the marginal cost MC functions on the same system of axes.

END OF PAPER