International Institute for Technology and Management

November 27th, 2007 **Unit: 05a – Mathematics 1**

GROUP(A)-VERSION A

This paper is not to be removed from the Examination Halls

Student Name :

Student Number :

Tuesday 27th November 13 : 30 pm – 15 : 30 pm

Candidates should answer **NINE** of the following **ELEVEN** questions:**SEVEN** from section A (60 marks in total) and **TWO** from section B (20 marks each).

Candidates are strongly advised to divide their time accordingly.

Graph paper is provided at the student request.

Calculators **May NOT** be used for this paper.

PLEASE TURN OVER

SECTION A

Answer all **SEVEN** questions from this section (60 marks in total)

1. The supply and the demand functions for a good are given ,respectively, by $\mathbf{q = 4p^2 + 2p - 1}$, $\mathbf{q = -4p^2 - 18p + 27}$ where p is the price.
Sketch the supply and the demand functions for $\mathbf{p} \geq 0$
Determine the equilibrium price and quantity.

2. The function f(x) is defined for x > 0 by:

$$f(x) = 2\ln(5x) - x^2$$

Find the stationary points of f(x) and specify their nature.

3. Determine the following integrals

$$\int (\ln x)^2 dx \quad , \int \frac{x^5}{\sqrt{2x^2+3}}\, dx$$

4. A firm has marginal cost qe^{q^2} and fixed costs are 5.
Find its total cost function.

5. The inverse demand function for a good takes the form

$p = \frac{a}{q+3}$ where a > 3 some fixed number.

The supply function is $p = q - 1$

Find expressions,in terms of a for the equilibrium price and quantity.

6. A firm's demand function is

$$p = mq + n \quad (m < 0 \; ; \; n > 0)$$

fixed costs are ***c*** and variable costs are ***v*** per unit.

Show that the profit is maximized when q = $\frac{v-n}{2m}$

7. Determine the following integrals

$$\int \frac{x-3}{x^2-6x+13}\, dx \quad , \quad \int \frac{dx}{\cos^2 x(\tan^2 x + 6\tan x + 8)}$$

SECTION B

Answer **TWO** questions from this section (20 marks each)

8.(a) A firm is a monopoly for the good it produces, It has a marginal cost function MC = $12q^2$ + 16 and fixed costs of 40.The demand equation for its good is given by P + 4q = 80 where p is the price. Find expressions in terms of q, for the total revenue and profit. Determine the value of q that maximises the profit.

(b) Determine the following integrals

$$\int \frac{dx}{x^{1/2} - x^{3/2}} \quad , \quad \int \frac{dx}{e^x - e^{-x}}$$

9. (a) A monopoly has fixed costs of **10** and Average variable cost function $\mathbf{q^2+4}$ .the demand equation for its product is **p+q =20**.Determine the profit function in terms of q. Determine also the production level that maximises the profit.

(b) At which values of x will the function

$$f(x) = \frac{x^7}{7} + \frac{x^6}{6} - \frac{2x^5}{5}$$

has a local maximum? Explain your answer.

10.(a) A firm's marginal revenue function is MR=11 – q
The firm's marginal cost function is
MC = q^2 – 3q + 3
where q is either the quantity sold or produced. Find the profit-maximizing level of output and verify that it is a maximum.

(b) Determine the following integrals

$$\int \frac{\ln x}{\sqrt{x}}\, dx \quad , \quad \int \frac{\sqrt{x-1}}{x-2}\, dx$$

11.A firm faces a total cost function TC = $2q^3$ – $8q^2$ + 24q

(i) Determine the firm's average cost (AC) and marginal cost (MC) functions.

(ii) Sketch the average cost (AC) and the marginal cost(MC) on the same graph.

(iii) If price is $ 15 , which level of output will a profit maximising firm choose?

END OF PAPER