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## International Institute for Technology and Management



## **Tutoring Sheet 7**

Unit 05a : Mathematics 1

## Answers

- 1. Find the derivative of the following functions:
- a.  $f(x) = (2x-3)^5$  is of the form U<sup>k</sup>, its derivative :kU<sup>k-1</sup>(U') f'(x) = 5(2x-3)^4 (2) = 10(2x-3)^4
- b.  $f(x) = \frac{5-3x}{4x-1}$  is of the form  $\frac{u}{v}$ , its derivative :  $\frac{u'v - v'u}{v^2} = \frac{-3(4x-1) - 4(5-3x)}{(4x-1)^2} = \frac{-17}{(4x-1)^2}$ c.  $f(x) = \frac{3}{x^2+1}$  is of the form  $\frac{u}{v}$ , its derivative :  $\frac{u'v - v'u}{v^2} = \frac{(0)(x^2+1) - (2x)(3)}{(4x-1)^2} = \frac{-6x}{(x^2+1)^2}$ d.  $f(x) = \frac{x^2 - 3x + 1}{x^2 + x - 2}$  is of the form  $\frac{u}{v}$ , its derivative :  $\frac{u'v - v'u}{v^2} = \frac{(2x-3)(x^2 + x - 2) - (2x+1)(x^2 - 3x + 1)}{(x^2 + x - 2)^2}$

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e. 
$$f(x) = x^2 e^x$$
 is of the form uv ,its derivative:  $u'v + v'u$   
 $f'(x) = 2xe^x + x^2e^x = (x^2 + 2x)e^x$   
f.  $f(x) = (x^2 - 1)\ln x$  is of the form uv ,its derivative:  $u'v + v'u$   
 $f'(x) = (2x)\ln x + (x^2 - 1)(\frac{1}{x}) = (2x)\ln x + \frac{x^2 - 1}{x}$   
g.  $f(x) = \frac{\ln x}{x}$  is of the form  $\frac{u}{v}$ , its derivative :  
 $\frac{u'v - v'u}{v^2} = \frac{(\frac{1}{x})(x) - (1)(\ln x)}{x^2} = \frac{1 - \ln x}{x^2}$   
h.  $f(x) = \frac{\sin x}{x}$  is of the form  $\frac{u}{v}$ , its derivative :  
 $\frac{u'v - v'u}{v^2} = \frac{(\cos x)(x) - (1)(\sin x)}{x^2} = \frac{x \cos x - \sin x}{x^2}$   
i.  $f(x) = x^2 \cos x$  is of the form uv ,its derivative:  $u'v + v'u$   
 $f'(x) = 2x\cos x + (-\sin x)(x^2) = 2x\cos x - x^2\sin x$   
j.  $f(x) = \sqrt{x^2 + 3}$  is of the form  $\sqrt{U}$  ,its derivative :  
 $\frac{U'}{2\sqrt{U}} = \frac{2x + 3}{2\sqrt{x^2 + 3}}$ 

k. 
$$f(x) = \ln(x^2 + x + 2)$$
 is of the form InU, its derivative :  

$$\frac{U'}{U} = \frac{2x + 1}{x^2 + x + 2}$$

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I. 
$$f(x) = \frac{e^x + 1}{e^x - 1}$$
 is of the form  $\frac{u}{v}$ , its derivative :

$$\frac{u'v - v'u}{v^2} = \frac{e^x(e^x - 1) - e^x(e^x + 1)}{(e^x - 1)^2} = \frac{e^{2x} - e^x - e^{2x} - e^x}{(e^x - 1)^2} = \frac{-2e^x}{(e^x - 1)^2}$$
  
m.  $f(x) = \frac{x^2 + 1}{\sqrt{3x - 1}}$  is of the form  $\frac{u}{v}$ , its derivative :  
 $\frac{u'v - v'u}{v^2} = \frac{(2x)(\sqrt{3x - 1}) - \frac{3(x^2 + 1)}{2\sqrt{3x - 1}}}{(\sqrt{3x - 1})^2}$   
 $= \frac{\frac{4x(3x - 1) - 3(x^2 + 1)}{2\sqrt{3x - 1}}}{3x - 1} = \frac{9x^2 - 4x - 3}{2(3x - 1)(\sqrt{3x - 1})}$   
n.  $f(x) = \ln(\frac{1 + x}{1 - x})$  is of the form  $\ln U$ , its derivative :  $\frac{U'}{U}$   
 $U = \frac{1 + x}{1 - x} \Rightarrow U' = \frac{2}{(1 - x)^2}$   
 $\frac{U'}{U} = \frac{\frac{2}{(1 - x)^2}}{\frac{1 + x}{1 - x}} = \frac{2}{(1 - x)(1 + x)}$ 

1-x

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2. Find the derivative of  $f(x) = (1+2x)e^{-x^2}$ find the value of x that makes f'(x) = 0is of the form uv ,its derivative: u'v + v'uu = 1 + 2x => u' = 2 $v = e^{-x^2} => v' = -2xe^{-x^2}$  $f'(x) = 2e^{-x^2} + (-2xe^{-x^2})(1+2x)$  $= 2e^{-x^2} - 2xe^{-x^2} - 4x^2e^{-x^2}$  $= 2e^{-x^2}(1 - x - 2x^2)$ 

 $F'(x)=0=>-2x^2-x+1=0\;$  a Quadratic equation with a=-2 ,  $b=-1\;$  and  $\;c=1\;$ 

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{9}}{-4} = \frac{1 \pm 3}{-4}$$
  
x = -1 or x =  $\frac{1}{2}$ 

**3.** Find the derivative of  $f(x) = x^2 - \ln(\sqrt{2}x)$  find the value of x that makes f'(x) = 0

$$f'(x) = 2x - \frac{\sqrt{2}}{\sqrt{2x}} = 2x - \frac{1}{x}$$

$$F(x) = \ln (cx)$$
$$=> F'(x) = \frac{1}{cx} \times c = \frac{1}{x}$$
Example:  $f(x) = \ln 3x$ 
$$=> f'(x) = \frac{1}{x}$$

$$f'(x) = 0 \implies 2x - \frac{1}{x} = 0 \implies 2x = \frac{1}{x} \implies 2x^{2} = 1$$
  
$$\Rightarrow x^{2} = \frac{1}{2} \implies x = \pm \frac{1}{\sqrt{2}}$$

## **End of Answers**