International Institute for Technology and Management

Tutoring Sheet 7

Unit 05a : Mathematics 1

Answers

1. Find the derivative of the following functions:

a. $f(x) = (2x-3)^5$ is of the form U^k ,its derivative : kU^{k-1} (U′)
f′(x) = $5(2x-3)^4$ (2) = $10(2x-3)^4$

b. $f(x) = \dfrac{5-3x}{4x-1}$ is of the form $\dfrac{u}{v}$, its derivative :

$$\frac{u'v - v'u}{v^2} = \frac{-3(4x-1)-4(5-3x)}{(4x-1)^2} = \frac{-17}{(4x-1)^2}$$

c. $f(x) = \dfrac{3}{x^2+1}$ is of the form $\dfrac{u}{v}$, its derivative :

$$\frac{u'v - v'u}{v^2} = \frac{(0)(x^2+1)-(2x)(3)}{(4x-1)^2} = \frac{-6x}{(x^2+1)^2}$$

d. $f(x) = \dfrac{x^2-3x+1}{x^2+x-2}$ is of the form $\dfrac{u}{v}$, its derivative :

$$\frac{u'v - v'u}{v^2} = \frac{(2x-3)(x^2+x-2)-(2x+1)(x^2-3x+1)}{(x^2+x-2)^2}$$

$$= \frac{4x^2-6x+5}{(x^2+x-2)^2}$$

e. $f(x) = x^2 e^x$ is of the form uv ,its derivative: u'v + v'u

f'(x) = $2xe^x + x^2e^x = (x^2 + 2x)e^x$

f. $f(x) = (x^2 - 1)\ln x$ is of the form uv ,its derivative: u'v + v'u

f'(x) = $(2x)\ln x + (x^2 - 1)\left(\frac{1}{x}\right) = (2x)\ln x + \frac{x^2 - 1}{x}$

g. $f(x) = \frac{\ln x}{x}$ is of the form $\frac{u}{v}$, its derivative :

$$\frac{u'v - v'u}{v^2} = \frac{\left(\frac{1}{x}\right)(x) - (1)(\ln x)}{x^2} = \frac{1 - \ln x}{x^2}$$

h. $f(x) = \frac{\sin x}{x}$ is of the form $\frac{u}{v}$, its derivative :

$$\frac{u'v - v'u}{v^2} = \frac{(\cos x)(x) - (1)(\sin x)}{x^2} = \frac{x\cos x - \sin x}{x^2}$$

i. $f(x) = x^2 \cos x$ is of the form uv ,its derivative: u'v + v'u

f'(x) = 2xcosx + (-sinx)(x^2) = 2xcosx – x^2sinx

j. $f(x) = \sqrt{x^2 + 3}$ is of the form $\sqrt{U}$,its derivative :

$$\frac{U'}{2\sqrt{U}} = \frac{2x + 3}{2\sqrt{x^2 + 3}}$$

k. $f(x) = \ln(x^2 + x + 2)$ is of the form lnU,its derivative :

$$\frac{U'}{U} = \frac{2x + 1}{x^2 + x + 2}$$

l. $f(x)=\dfrac{e^x+1}{e^x-1}$ is of the form $\dfrac{u}{v}$, its derivative :

$$\frac{u'v-v'u}{v^2}=$$

$$\frac{e^x\left(e^x-1\right)-e^x\left(e^x+1\right)}{\left(e^x-1\right)^2}=\frac{e^{2x}-e^x-e^{2x}-e^x}{\left(e^x-1\right)^2}=\frac{-2e^x}{\left(e^x-1\right)^2}$$

m. $f(x)=\dfrac{x^2+1}{\sqrt{3x-1}}$ is of the form $\dfrac{u}{v}$, its derivative :

$$\frac{u'v-v'u}{v^2}=\frac{(2x)\left(\sqrt{3x-1}\right)-\dfrac{3(x^2+1)}{2\sqrt{3x-1}}}{(\sqrt{3x-1})^2}$$

$$=\frac{\dfrac{4x(3x-1)-3(x^2+1)}{2\sqrt{3x-1}}}{3x-1}=\frac{9x^2-4x-3}{2(3x-1)(\sqrt{3x-1})}$$

n. $f(x)=\ln\left(\dfrac{1+x}{1-x}\right)$ is of the form lnU,its derivative : $\dfrac{U'}{U}$

$$U=\frac{1+x}{1-x}\Rightarrow U'=\frac{2}{(1-x)^2}$$

$$\frac{U'}{U}=\frac{\dfrac{2}{(1-x)^2}}{\dfrac{1+x}{1-x}}=\frac{2}{(1-x)(1+x)}$$

2. Find the derivative of $f(x) = (1+2x)e^{-x^2}$
find the value of x that makes $f'(x) = 0$
is of the form uv ,its derivative: u′v + v′u
u = 1 + 2x => u′ = 2
v = e^{-x^2} => v′ = -2xe^{-x^2}
f′(x) = 2e^{-x^2} + (-2xe^{-x^2})(1+2x)
= 2e^{-x^2} -2xe^{-x^2} -4x^2e^{-x^2}
= 2e^{-x^2}(1 – x – 2x^2)

F′(x) = 0 => -2x^2 – x + 1 = 0 a Quadratic equation with
a = - 2 , b = -1 and c = 1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{9}}{-4} = \frac{1 \pm 3}{-4}$$

x = -1 or x = ½

3. Find the derivative of $f(x) = x^2 - \ln(\sqrt{2}x)$
find the value of x that makes $f'(x) = 0$

$$\mathbf{f'(x)} = 2x - \frac{\sqrt{2}}{\sqrt{2}x} = 2x - \frac{1}{x}$$

f′(x) = 0 => $2x - \frac{1}{x} = 0$ => 2x = $\frac{1}{x}$ => 2x^2 = 1

⇨ x^2 = $\frac{1}{2}$ => x = $\pm\frac{1}{\sqrt{2}}$

F (x) = ln (cx)
=> F'(x) = $\frac{1}{cx} \times c = \frac{1}{x}$
Example: f(x) = ln 3x
=> f'(x) = $\frac{1}{x}$

End of Answers