



Unit 05a: Mathematics 1 **Tutoring Sheet #4**
Basics IV – Logarithms/Exponential
Solution

- Translate each logarithmic statement into an equivalent exponential statement:
 - $\log 1000 = 3 \Rightarrow 10^3 = 1000$
 - $\log 0.01 = -2 \Rightarrow 10^{-2} = 0.01$
 - $\log_3 81 = 4 \Rightarrow 3^4 = 81$
 - $\log_2 \frac{1}{4} = -2 \Rightarrow 2^{-2} = \frac{1}{4}$
 - $\ln e = 1 \Rightarrow e^1 = e$
- Translate each exponential statement into an equivalent logarithmic statement:
 - $3^{-2} = 1/9 \Rightarrow \log_3 1/9 = -2$
 - $16^{1/2} = 4 \Rightarrow \log_{16} 4 = 1/2$
 - $10^{1.8751} = 75 \Rightarrow \log 75 = 1.8751$
 - $e^{3.2189} = 25 \Rightarrow \ln 25 = 3.2189$
- Without using a calculator, evaluate each of the following:
 - $\log 100 = \log 10^2 = 2 \log 10 = 2(1) = 2$
 - $\log 0.001 = \log 10^{-3} = -3 \log 10 = -3(1) = -3$
 - $\log_5 25 = \log_5 5^2 = 2 \log_5 5 = 2(1) = 2$
 - $\log_3 1/27 = \log_3 3^{-3} = -3 \log_3 3 = -3(1) = -3$
 - $\ln 1/e = \ln e^{-1} = -1 \ln e = -1$
 - $\ln \sqrt{e} = \ln e^{1/2} = 1/2 \ln e = 1/2$
- Express each of the following in terms of $\ln x$ or $\ln y$ only:
 - $\ln(x^2 y^5) = \ln x^2 + \ln y^5 = 2 \ln|x| + 5 \ln y$
 - $\ln(\sqrt{x} y) = \ln \sqrt{x} + \ln y = \ln x^{1/2} + \ln y = 1/2 \ln x + \ln y$
 - $\ln(\sqrt{x}/y) = \ln \sqrt{x} - \ln y = \ln x^{1/2} - \ln y = 1/2 \ln x - \ln y$
- Solve each of the following equations :
 - $\log_3 (5x+1) = 2 \Rightarrow 3^2 = 5x + 1 \Rightarrow x = 8/5$
 - $\log x - \log (x+3) = -1 \Rightarrow \log (x/x+3) = -\log 10$
 $\Rightarrow \log (x/x+3) = \log 10^{-1} \Rightarrow x / x+3 = 1/ 10$
 $\Rightarrow 10x = x + 3 \Rightarrow x = 1/3$
 - $\log(x-1) + \log(x+2) = 1 \Rightarrow \log(x-1)(x+2) = \log 10$
 $\Rightarrow (x-1)(x+2) = 10 \Rightarrow x^2 + x - 12 = 0$
 $\Rightarrow x = 3 ; x = -4$ which is rejected since $x > 1 ; x > -2$

Recall : $\ln x = a \Rightarrow x = e^a$

$e^x = a \Rightarrow x = \ln a$

4. $\ln(2x - 3) = 2 \Rightarrow 2x - 3 = e^2 \Rightarrow x = (e^2 + 3) / 2$

5. $e^{3x - 2} = 5 \Rightarrow 3x - 2 = \ln 5 \Rightarrow x = (2 + \ln 5) / 3$

6. $e^x + e^{-x} = -2 \Rightarrow e^x + 1/e^x = -2 \Rightarrow e^{2x} + 1 = -2e^x$
 $\Rightarrow e^{2x} + 2e^x + 1 = 0 \Rightarrow (e^x + 1)^2 = 0$
 $\Rightarrow e^x + 1 = 0 \Rightarrow e^x = -1$ No solution since $e^x > 0$

7. $e^{2x} - e^x = 0 \Rightarrow e^x(e^x - 1) = 0 ; e^x \neq 0 \Rightarrow e^x = 1$
 $\Rightarrow x = \ln 1 = 0$

8. $4^{x+2} = 2^{x-1} \Rightarrow (2^2)^{x+2} = 2^{x-1} \Rightarrow 2^{2x+4} = 2^{x-1}$
 $\Rightarrow 2x + 4 = x - 1 \Rightarrow x = -5$

9. $10e^{3x-5} = 7 \Rightarrow e^{3x-5} = 7/10 \Rightarrow 3x - 5 = \ln(0.7)$
 $\Rightarrow x = (5 + \ln 0.7) / 3$

10. $3^x = 5 \Rightarrow x \ln 3 = \ln 5 \Rightarrow x = \ln 5 / \ln 3$

11. $(\log x)^2 - \log x^2 = 0 \Rightarrow (\log x)^2 - 2 \log x = 0$
 $\Rightarrow \log x(\log x - 2) = 0 \Rightarrow \log x = 0 \Rightarrow x = 10^0 = 1$
or $\log x - 2 = 0 \Rightarrow \log x = 2 \Rightarrow x = 10^2 = 100$

12. $2^x = 3^{x-1} \Rightarrow x \ln 2 = (x-1) \ln 3 \Rightarrow x \ln 2 = x \ln 3 - \ln 3$
 $\Rightarrow x \ln 2 - x \ln 3 = -\ln 3 \Rightarrow x(\ln 2 - \ln 3) = -\ln 3$
 $\Rightarrow x = -\ln 3 / (\ln 2 - \ln 3)$

13. $\ln(4x-2) = \ln 4 - \ln(x-2) \Rightarrow \ln(4x-2) + \ln(x-2) = \ln 4$
 $\Rightarrow \ln(4x-2)(x-2) = \ln 4 \Rightarrow (4x-2)(x-2) = 4$
 $\Rightarrow 4x^2 - 10x + 4 = 4 \Rightarrow 4x^2 - 10x \Rightarrow 2x(2x - 5) = 0$
 $\Rightarrow x = 0 ; x = 5/2$

14. $\log(-x) = -4 \Rightarrow 10^{-4} = -x \Rightarrow x = -10^{-4}$

15. $\log(x+4) - \log(x+2) = \log x \Rightarrow \log(x+4/x+2) = \log x$
 $\Rightarrow x+4 / x+2 = x \Rightarrow x^2 + 4x = x + 2 \Rightarrow x^2 + 3x - 2 = 0$
 $\Rightarrow x = \frac{-3 \pm \sqrt{17}}{2}$

16. $e^{x^2} = 100 \Rightarrow x^2 \ln e = \ln 100 \Rightarrow x^2 = \ln 100 \Rightarrow x = \pm \sqrt{\ln 100}$

17. $2^{x-8} = 32 \Rightarrow 2^{x-2} = 2^5 \Rightarrow x-2 = 5 \Rightarrow x = 7$

18. $3^{2x-1} = 1/81 \Rightarrow 3^{2x-1} = 3^{-4} \Rightarrow 2x-1 = -4 \Rightarrow x = -3/2$