



Tutoring Sheet #2 - Solution

Unit 05a : Mathematics 1

1. Solve each of the following equations/inequalities:

$$1. 9(3x-4) - 2x = 11 + 5(4x - 1) \Rightarrow 27x - 36 - 2x = 11 + 20x - 5 \\ \Rightarrow 25x - 36 = 20x + 6 \Rightarrow 5x = 42 \Rightarrow x = 42/5 \Rightarrow x = 8.4$$

$$2. \frac{x}{3} - 16 = \frac{x}{12} + 14 \Rightarrow x/3 - x/12 = 14 + 16 \Rightarrow 3x/12 = 30 \Rightarrow x = 120$$

$$3. \frac{5}{x} + \frac{3}{x+4} = \frac{7}{x} \Rightarrow \frac{8x+20}{x(x+4)} = \frac{7}{x} \Rightarrow x(8x+20) = 7x(x+4)$$

$$8x^2 + 20x = 7x^2 + 28x \Rightarrow x^2 - 8x = 0 \Rightarrow x(x-8) = 0 \Rightarrow x = 0 ; x = 8$$

$$4. |2x - 3| = 7 \Rightarrow 2x - 3 = \pm 7 \Rightarrow x = 5 ; x = -2$$

$$5. |x| + |x + 3| = -5 \text{ No real solution since } | | > 0$$

$$6. |5x+2| - 3 < 4 \Rightarrow |5x+2| < 7 \Rightarrow -7 < 5x+2 < 7 \Rightarrow -9/5 < x < 1$$

$$7. |3x - 5| + 2 > 10 \Rightarrow |3x - 5| > 8 \Rightarrow 3x - 5 < -8 \Rightarrow x < -1 \text{ or}$$

$$3x - 5 > 8 \Rightarrow x > 13/3$$

$$8. |x^2 - 1| = 8 \Rightarrow x^2 - 1 = \pm 8 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3 \text{ or } x^2 = -7 \text{ No solution}$$

$$9. (x+1)^2 = 7 \Rightarrow x+1 = \pm \sqrt{7} \Rightarrow x = \sqrt{7} - 1 ; x = -\sqrt{7} - 1$$

$$10. 2x^2 - 3x = 0 \Rightarrow x(2x-3) = 0 \Rightarrow x = 0 ; x = 3/2$$

$$11. 4x^2 - 12x + 9 = 0 \Rightarrow (2x-3)^2 = 0 \Rightarrow x = 3/2$$

$$12. 5x^2 + 2x + 1 = 0 ; b^2 - 4ac = 4 - 4(5)(1) = -16 < 0 \text{ no real roots.}$$

$$13. -3x^2 + 5x + 2 = 0 ; \text{ using quadratic formula: } x = 2 ; x = -1/3$$

$$14. -x^4 + 6x^2 - 5 = 0 ; \text{ using quadratic formula: } x^2 = 1 \Rightarrow x = \pm 1$$

$$\text{or } x^2 = 5 \Rightarrow x = \pm \sqrt{5}$$

$$15. 2x^4 = 7x^2 + 15 \Rightarrow 2x^4 - 7x^2 - 15 = 0 \Rightarrow x^2 = 5 \Rightarrow x = \pm \sqrt{5}$$

$$\text{or } x^2 = -3/2 \text{ no real solution.}$$

$$16. \sqrt{x-2} = 3 - 5x \Rightarrow x-2 = (3-5x)^2 \Rightarrow 25x^2 - 31x + 11 = 0 ; \text{ No roots.}$$

$$17. x^2 - 2 = \sqrt{-x+6} \Rightarrow (x^2-2)^2 = x+6 \Rightarrow x^4 - 4x^2 + 4 = -x+6$$

$$\Rightarrow x^4 - 4x^2 + x - 2 = 0 \Rightarrow x^2(x^2-4) + x - 2 = 0 \Rightarrow x(x-2)(x+2) + x - 2 = 0$$

$$\Rightarrow (x-2)(x^2+2x+1) = 0 \Rightarrow x = 2 ; x = -1$$

$$18. x^3 - 1 = 0 \Rightarrow (x-1)(x^2-x+1) = 0 \Rightarrow x = 1 ; x^2-x+1 = 0 \text{ no roots.}$$

$$19. x^3 - 3x + 2 = 0 ; x = 1 \text{ double root ; } x = -2 ;$$

$$20. x + \frac{1}{x} = -2 \Rightarrow x^2 + 1 = -2x \Rightarrow x^2 + 2x + 1 = 0 \Rightarrow (x+1)^2 = 0$$

$$\Rightarrow x = -1 .$$

2. Solve each of the following simultaneous equations:

$$(1.) -13x + 6y + 132 = 0 \text{ -----(1)}$$

$$x + 3y - 69 = 0 \text{ -----(2)}$$

Multiply (2) by -2 to get rid of y :

$$-2x - 6y + 138 = 0$$

$$-13x + 6y + 132 = 0 \text{ Adding these two:}$$

$$-15x + 270 = 0 \Rightarrow x = 18 \text{ substituting this in (2) :}$$

$$18 + 3y - 69 = 0 \Rightarrow y = 17$$

$$(2.) \mathbf{q} = \frac{2}{3}p - \frac{16}{3}$$

$$\mathbf{q} = \mathbf{20} - \mathbf{2p}$$

Equating both equations : $q = q$

$$\frac{2}{3}p - \frac{16}{3} = \mathbf{20} - \mathbf{2p} \text{ multiply by 3}$$

$$2p - 16 = 60 - 6p \Rightarrow p = 76/8 = 19/2$$

$$q = 20 - 2p = 20 - 19 = 1$$

$$(3.) 6x^2 - 12y = 0 \text{ ; } -12x + 2y = 0 \Rightarrow y = 6x \text{ substitute in}$$

$$\text{the other equation: } 6x^2 - 12(6x) = 0$$

$$\Rightarrow 6x^2 - 72x = 0 \Rightarrow 6x(x-12)=0 \Rightarrow x = 0 \text{ ; } x = 12$$

$$x=0 \Rightarrow y=6x=6(0) = 0$$

$$x=12 \Rightarrow y = 6x=6(12) = 72.$$

$$(4.) p_y - p_x + 13 - 6x = 0$$

$$p_x - 4p_y + 26 - 6y = 0 \text{ ; In part (4) solve for } p_x \text{ and } p_y$$

Add these two to get rid of p_x :

$$-3p_y + 39 - 6x - 6y = 0 \Rightarrow p_y = \frac{6x + 6y - 39}{-3}$$

$$p_y = -2x - 2y + 13 \text{ ; substitute this in the first}$$

$$\text{equation, } p_x = -2x - 2y + 13 + 13 - 6x$$

$$p_x = -8x - 2y + 36$$