



Tutoring Sheet #1 - Solution

Unit 05a : Mathematics 1

1. Simplify each of the following. Leave answers with exponents:

a. $3^8 \times 3^3 = 3^{11}$ b. $(-3)^5 \times 3^4 = -3^5 \times 3^4 = -3^9$

c. $(2x)^5 \times (2x)^6 = 2^5x^5 \times 2^6x^6 = 2^{11}x^{11}$

d. $(2xy^2z^3)^4 = 2^4x^4y^8z^{12}$ e. $\frac{-12x^3y^2z^5}{6x^2y^2z^6} = -2xz^{-1}$

f. $3\sqrt{y} \times 5\sqrt{y} = 15y$ g. $\sqrt[3]{x^2y^3} = x^{2/3}y$

h. $\frac{5}{\sqrt{5}} = 5 \times 5^{-1/2} = 5^{1/2}$ i. $\left(\frac{ab^2}{a+b}\right)^0 = 1$ j. $\frac{x^4 \times x^{-3}}{(x^{-2})^3} = \frac{x}{x^6} = x^{-5}$

2. Add or subtract as indicated :

a. $3x^2 + 2x^2 - 5x + 4x^3 + x^2 - 8x = 4x^3 + 6x^2 - 13x$

b. $3y^3 + 9y^2 - 11y + 8 + 4y^2 - 10y + 6 = 3y^3 + 13y^2 - 21y + 14$

c. $6x - x^2 + 16y - 2y^2 - 0.5x^2 - 0.5y^2 - xy = -1.5x^2 - 2.5y^2 + 6x + 16y - xy$

3. Perform each of the following operations:

a. $x(13-2x-y) + y(13-x-2y) = -2x^2 + 13x - 2y^2 - 2xy + 13y$

b. $(3x+5)(4x^2-2x-1) = 12x^3 + 14x^2 - 13x - 5$

c. $(3x-1)^2 = 9x^2 - 6x + 1$

d. $(x-2)(2x+3)^2 = (x-2)(4x^2+12x+9) = 4x^3 + 4x^2 - 15x - 18$

e. $x(x-4)^3 = x(x^3-12x^2+48x-64) = x^4 - 12x^3 + 48x^2 - 64x$

f. $(2p-5q)(2p+5q) = (2p)^2 - (5q)^2 = 4p^2 - 25q^2$

4. Factor as completely as possible :

a. $2xy^2 - 4xy + 5x = x(2y^2 - 4y + 5)$

b. $x^2 - 6x + 5 = (x-1)(x-5)$

c. $3x^4 + 13x^3 + 4x^2 = x^2(3x^2 + 13x + 4) = x^2(3x+1)(x+4)$

d. $10x^2 - 11x + 3 = (5x-3)(2x-1)$

e. $4x^2 - 20x + 25 = (2x-5)(2x-5) = (2x-5)^2$

f. $81p^2 - 25q^2 = (9p-5q)(9p+5q)$; using $a^2 - b^2$

g. $8p^3 - 1 = (2p)^3 - 1 = (2p-1)(4p^2 - 2p + 1)$; using $a^3 - b^3$

h. $125p^3 + 216 = (5p)^3 + 6^3 = (5p+6)(25p^2 - 60p + 36)$; using $a^3 + b^3$

5. Perform each operation :

a. $\frac{4x}{5} \times \frac{35x}{12} = \frac{7x^2}{3}$

$$b. \frac{5x^2}{24} - \frac{75x}{36} = \frac{15x^2 - 150x}{72}$$

$$c. \frac{6}{15x} + \frac{2}{3x} - \frac{9}{10x} = \frac{12 + 20 - 27}{30x} = \frac{5}{30x} = \frac{1}{6x}$$

$$d. \frac{x^2 - 3x + 2}{2x(x-1)} \div \frac{x-2}{8x} = \frac{(x-1)(x-2)}{2x(x-1)} \times \frac{8x}{x-2} = 8/2 = 4$$

e.

$$\frac{2x-10}{5x} - \frac{20x-25}{12} = \frac{24x-120-100x^2+125x}{60x} = \frac{-100x^2+149x-120}{60x}$$

$$f. \frac{5}{x-2} - \frac{4}{x} = \frac{5x-4(x-2)}{x(x-2)} = \frac{5x-4x+8}{x(x-2)} = \frac{x+8}{x(x-2)}$$

6. a. Simplify : i. $\sqrt[3]{54p^3q^5} = \sqrt[3]{2 \times 27p^3q^3q^2} = 3pq\sqrt[3]{2q^2}$

ii. $3^{-2} + 3^{-1} = 1/3^2 + 1/3 = 1/9 + 1/3 = 4/9$

iii. $\sqrt[5]{-32} = \sqrt[5]{(-2)^5} = -2$ iv. $\frac{\sqrt{2}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1+\sqrt{3}} = \frac{2(1-\sqrt{3})}{1-3} = \sqrt{3} - 1$

v. $((2\sqrt{5} - \sqrt{3})(\sqrt{5} + 2\sqrt{3})) = 2(5) + 4\sqrt{15} - \sqrt{15} - 2(6) = -2 + 3\sqrt{15}$

vi. $p^{\frac{2}{3}} \left(2p^{\frac{1}{3}} + 5p \right) = 2p^{\frac{2}{3} + \frac{1}{3}} + 5p^{\frac{2}{3} + 1} = 2p + 5p^{5/3}$

b. Estimate each of the following : $5\sqrt{7}, \sqrt{22} + 3, \sqrt{40}, \sqrt{93}, \sqrt{897}$

1. $5\sqrt{7}$: 7 is between $4=2^2$ and $9 = 3^2$; $\sqrt{7}$ between 2 and 3
 $5\sqrt{7}$ between 10 and 15.

2. $\sqrt{22} + 3$: 22 is between $16=4^2$ and $25=5^2$;
 $\sqrt{22}$ between 4 and 5 ; $\sqrt{22} + 3$ between 7 and 8 .

3. $\sqrt{93}$: 93 between $81=9^2$ and $100 = 10^2$
 $\sqrt{93}$ between 9 and 10 .

4. $\sqrt{897}$: $897 < 900$; is close to $900 = 30^2$

$\sqrt{897}$ is a little bit less than 30

$\sqrt{897}$ is 29.something i.e between 29 and 30 .