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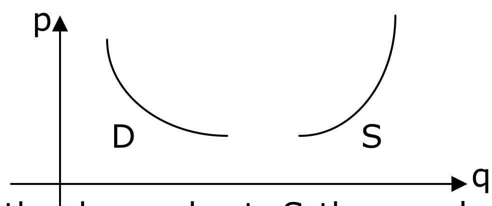
International Institute for Technology and Management

Unit 05a : Mathematics 1 **Handout #6**

Mathematical Economic Models

Topic	Interpretation
Supply and Demand (q , p) Example: (2000, 5)	p:price per unit of the good . q:quantity of the good in the market. 2000 units available at the price of $5 per unit.
Horizontal axis : q-axis Vertical axis : p-axis p:selling price,q: demand q is the quantity which would be sold to customers at that price. *N.B. : you may choose the horizontal axis as the p-axis if q is given as a function of p such as : q = 3p + 5*	 D: the demand set- S:the supply set
Demand Function q^D is the quantity which would be sold if the price were p. <u>Example1</u> : (q,p)=(30,5) $q^D(5) = 30$ <u>Example2</u>:Suppose the demand set D consists of the points (q,p) on the straight line : 6q+8p = 125	Example 2: The demand function q^D: find q in terms of p : 6q+8p = 125 6q= 125 – 8p q = $\frac{125-8p}{6}$ so, $q^D = \frac{125-8p}{6}$ The Inverse demand function p^D: find p in terms of q :$p^D = \frac{125-6p}{8}$
Supply Function Supply set S consisting of (q,p) q:the amount supplied to the market if the price were p. <u>Example:</u> Suppose the supply set S consists of the points (q,p) on the straight line : 2q - 5p = - 12	The supply function q^S: find q in terms of p : 2q-p = -12 ; $q^S = \frac{5p-12}{2}$ The Inverse supply function p^S: find p in terms of q : $p^S = \frac{2q+12}{5}$

Market Equilibrium Where the quantity supplied is exactly balanced by the quantity required: $q^D = q^S$ Example: Suppose the sets D and S are respectively the pairs (q , p) satisfying : q+5p = 40 ; 2q – 15p = -20	The equilibrium set E is the single pair (20,4) obtained upon solving the simultaneous equations: q +5p = 40 2q – 15p = -20 We usually denote them by : $q_0 = 20$; $p_0 = 4$
Cost Analysis **Cost Function** TC = VC + FC FC = TC(0) Example1: The total Cost function of Manufacturing an item is : TC = 12q + 100 Find the fixed cost and the cost of producing 6 items. **Marginal cost MC** Is the rate of change of cost, Cost of Making one more item after q has been made. If the total cost is TC = *m*x + b then its graph is a straight line of slope *m*,since the slope is the rate of change, the marginal cost is **m**. Example2:The marginal cost of TC = 25q + 200 is MC = 25 **Average Cost AC** $AC = \frac{TC}{q}$ Example3**:**the average cost function of an item is : $AC = \mathbf{q} + \frac{1}{q^2} + \frac{\ln(1+q^2)}{q}$ Find the total cost.	TC : Total cost VC :Variable cost FC : Fixed cost, cost before starting production: producing 0 items(q=0) FC = TC(0) FC : Design product, setup factory… VC : Labor, materials, packing shipping… Example1 FC = TC(0)= 12(0) + 100 = 100 Cost of producing 6 items TC(6)= 12(6) + 100 = 172 Marginal cost is used to make decisions in areas such as cost control, pricing and production planning. Example2: Suppose q items are produced ,then the cost of producing one more item(i.e. producing q+1) is the cost of producing q+1 items – the cost of producing q items = TC(q+1) – TC(q) = 25(q+1) + 200 – 25q – 200 = 25 Average cost is the total cost divided by the quantity produced Example3 $AC = \frac{TC}{q} \Rightarrow TC = q \times AC$ $TC = q\left(\mathbf{q} + \frac{1}{q^2} + \frac{\ln(1+q^2)}{q}\right)$ $TC = q^2 + 1 + \ln(1+q^2)$

Average Variable Cost AVC TC = VC + FC $\Rightarrow$VC = TC – FC AVC = $\frac{VC}{q} = \frac{TC-FC}{q}$ Example4: A company's total cost function is : TC = $5q - 2q^2 + 3q^3 + 20$ Its fixed cost is 20. Find the average variable cost.	Example4: AVC = $\frac{VC}{q} = \frac{TC-FC}{q}$ AVC = $\frac{5q-2q^2+3q^3+20-20}{q}$ AVC = $\frac{5q-2q^2+3q^3}{q}$ AVC = $5 - q + 3q^2$
Profit $\prod$ = TR – TC --(1) ***Profit = Revenue – Cost***	$\prod$: The profit TR : total Revenue from producing q units . TC : total Cost to produce q units.
Total Revenue Is the product of the quantity q produced and the price p it holds. Price x quantity TR = q x p --(2)	p : Price per unit . If 10 items are produced and sold at the price of 5 ,then the total revenue = 10x5 = 50
(1) & (2): $\prod$ (q) =pq – TC	The best value of q is that which maximizes the profit.
Break Even When the profit is **0**	$\prod$ = TR – TC = 0 TR = TC
Competition versus Monopoly ***Perfect competition*** When the firm is small and its output (goods) does not affect the market price of its good. ***Monopoly*** Where the firm supplies the entire quantity of the good. The price is determined by the demand set D.	The market price is not under the firm's control. The price = p^D: the inverse demand function. The price the consumers will pay when q is available.