For comments, corrections, etc…Please contact Ahnaf Abbas: ahnaf@mathyards.com

International Institute
for Technology and Management

Unit 05a: Mathematics I **Handout #20**

Applications of Matrices

Topic	Interpretation
Matrix Definition A matrix is an array of numbers: **A** = $\begin{pmatrix} a_{11} & a_{12} & ..... & a_{1n} \\ a_{21} & a_{22} & ..... & a_{2n} \\ . & . & . & . \\ a_{m1} & a_{m2} & ...... & a_{mn} \end{pmatrix}$ Matrices are denoted by capital letters : **A,B,C,…..** Matrix size or rank is determined by the number of rows × the number of columns it has. We say **A** has **m** rows and **n** columns or it is an **m**×**n** matrix. **Square Matrix** A matrix with the same number of rows as columns:2×2 , 3×3,4×4 are all square matrices. **Identity Matrix** Has **1** in each of the positions in the main diagonal and **0** elsewhere. **Note that** : **I** is a Square matrix. **Matrix Addition** If **A** and **B** are two matrices of the same size then we define **A+B** to be the matrix whose elements are the sums of the corresponding elements in **A** and **B**. **Only matrices of the same size can be added.** **A + (B + C) = (A+B)+C** **A – B = A + (-B)** **k(A+B) = kA + kB**	<u>*Example1*</u>*:* **A** = $\begin{pmatrix} 2 & 0 \\ -5 & 3 \end{pmatrix}$ is **2** × **2** **B** = $\begin{pmatrix} 3 & 0 & -1 \\ 6 & 8 & 2 \\ 1 & 0 & 7 \\ -5 & -1 & 4 \end{pmatrix}$ is **4** × **3** **C** = $\begin{pmatrix} 1 & 6 & 5 & -2 & 3 \end{pmatrix}$ is **1** × **5** <u>*Example2*</u>*:* **I** = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the **2** × **2** Identity matrix **I** = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is the **3** × **3** Identity matrix <u>*Example3*</u>*:* $\begin{pmatrix} 2 & 5 & -1 \\ 0 & 3 & 7 \\ -9 & 1 & -6 \\ 3 & 0 & 5 \end{pmatrix} + \begin{pmatrix} 5 & 4 & 1 \\ 1 & 5 & 4 \\ 4 & 5 & 1 \\ 1 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 7 & 9 & 0 \\ 1 & 8 & 11 \\ -5 & 6 & -5 \\ 4 & 4 & 10 \end{pmatrix}$

Matrix Multiplication
For the product of two matrices **A** and **B** to be defined, the number of columns of **A** must be the same as the number of rows in **B**:
A : $m \times \mathbf{n}$; B : $\mathbf{n} \times p$ then **AB** is defined and of rank $\mathbf{m} \times \mathbf{p}$

Properties
For any matrices **A , B , C** such that all the indicated sums and products exist:
A(BC) = (AB)C
A(B+C) = AB + BC

Remark
In general, AB may not equal BA.

Example5: suppose **A** is 2x**3** and **B** is **3**x5 then **AB** is defined**,** but **BA** is not defined.
B(3x**5**) , A(**2**x3)

Example4:

$$\begin{pmatrix} 2 & 3 & 1 \\ 4 & 2 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 & 4 & 3 \\ 2 & 3 & 5 & 1 \\ 6 & 7 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 14 & 20 & 26 & 17 \\ 38 & 49 & 41 & 24 \end{pmatrix}$$

The product is obtained by multiplying each row of **A** by the columns of **B**(first by first and so on) , The first entry:

$$\begin{pmatrix} 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} = 2x1 + 3x2 + 1x6 = 14$$

The second entry:

$$\begin{pmatrix} 2 & 3 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix} = 2x2 + 3x3 + 1x7 = 20$$

and so on ……

Solving A system of three equations with three unknowns

when they ask for Matrix method ,you **can not** use Algebra , substitution , manipulation etc...
For example :
x+y+z = 6 ---------(1)
2x - y + z = 3 ----- (2)
x + z = 4 ---------- (3)
From (1) : z = 6 - x - y
Substitute for z in (2) and (3) :
(2): 2x - y + 6 - x - y = 3 implies x - 2y = -3 ----(4)
(3): x + 6 - x - y = 4 implies y = 2
putting this in (4) : x - 2(2) = - 3 implies x = 1
Finally : z = 6 - x - y = 6 - 1 - 2 = 3
Hence (x,y,z) = (1,2,3)

Using the above method will result in ZERO Mark if the the required is :Using Matrix Method

Instead ,you need to construct the matrix (Read slowly and carefully, don't move to another step before understanding the previous one):

$$\begin{pmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 1 & 3 \\ 1 & 0 & 1 & 4 \end{pmatrix}$$

1 1 1 | 6 --------R1
2 -1 1| 3 --------R2
1 0 1 | 4 --------R3
You need to make it(Note the three zeros) :
1 ? ? | ?
0 1 ? | ?
0 0 ? | ?
The normal procedure is to manipulate this in **3 steps** :
R1 with R2 ; R1 with R3 ; R2 with R3
Our aim is to:
1.)Make the first element of the second row (which is 2) a zero (this can be done by playing with R1 and R2)
2.)Make the first element in the third row (which is 1) a zero(this can be done by playing with R1 and R3)
3.)Make the second element of the third row a zero we need to play with R2 and R3
Step1 : **R1 with R2** :
We do this by multiplying R1 by - 2 and then add it to R2 :
-2R1 + R2 : 0 - 3 - 1 | - 9
The Matrix becomes:
1 1 1 | 6 --------R1
0 -3 -1|- 9--------R2
1 0 1 | 4 --------R3
Note that we only replaced R2 ; R1 is left un tampered.
Step2 : **R1 with R3** :
to make the first element in the third row (which is 1) ,
it is enough to calculate :
-R1 + R3 : 0 -1 0 | -2
The matrix becomes :
1 1 1 | 6 --------R1
0 -3 -1 |- 9--------R2
0 -1 0 | -2 --------R3
Step3 : **R2 with R3**
to make the second element of the third row (which is - 1)
a zero
R2 - 3R3 : 0 0 -1 | - 3
The Matrix becomes :

1 1 1 | 6 --------R1
0 -3 -1 |- 9--------R2
0 0 -1 | -3 --------R3
Now the last row means :
0x + 0y - z = -3 implies z = 3
Second row :
0x - 3y - z = - 9 , substitute for z (this is called back-substitution):
-3y - 3 = - 9 implies y = 2
First row :
x + y + z = 6 implies x + 2 + 3 = 6 implies x = 1
Hence (x,y,z) = (1,2,3)

Special cases :

(1) Case of impossible solution : we say the system is ***inconsistent***

This occurs when the last row in the final matrix looks like:

0 0 0 | N where N is any number.

In this case 0 X z = N which is impossible

(2) Case of infinite number of solutions:

This occurs when the last row in the final matrix looks like:

0 0 0 | 0

In this case 0 X z = 0 , here z could be any real number , and the system has infinite number of solution
Assume z = t (t any real number)
e.g.

1 1 1 | 3
0 1 2 | 0
0 0 0 | 0

Here 0 X z = 0 , let z = t
Row2: y + 2z = 0 implies y = -2z = -2t
Row1: x +y + z = 3 implies x -2t + t = 3 , x = t +3
Hence (x,y,z) = (t+3,-2t,t) where t is any real number.