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International Institute
for Technology and Management

Unit 05a : Mathematics 1 **Handout #2**

Basics II: Equations

Topic	Interpretation
Linear equations In which all variables are raised to the power of one: It can be solved by moving the unknown to left side and all other terms to the right side: ax + b =0 $\Rightarrow$ ax = -b $\Rightarrow$ $x = \frac{-b}{a}$ provided a $\neq 0$.	Example1: $\frac{x}{4} - 3 = \frac{x}{5} + 1$ a good practice is to get rid of the fractions : Multiplying both sides by 20 : 5x – 60 = 4x + 20 $\Rightarrow$ 5x – 4x = 20 + 60 $\Rightarrow$ x = 80 Example2: 3(2x -1) + 5x + 7 = -2x + 43 6x – 3 + 5x + 7 = -2x + 43 11x + 4 = -2x + 43 $\Rightarrow$ 11x + 2x =43 – 4 13x = 39 $\Rightarrow$ x = 39/3 $\Rightarrow$ x = 3
Absolute Value Equations |x| = a ; a ≥ 0 (since |x| ≥ 0 **)** **|x| = a $\Rightarrow$ x = $\pm$ a** Remove the absolute value and $\pm$ the answer	Example1: | x - 3 | = 5 $\Rightarrow$ x – 3 = $\pm$ 5 Either x – 3 = - 5 $\Rightarrow$ x = -5 + 3 = -2 OR x – 3 = 5 $\Rightarrow$ x = 5 + 3 = 8 Example2: | x | = - 7 ;No solution since |x| ≥ 0 **.**
Absolute Value inequalities |x| < a $\Rightarrow$ - a < x < a |x| > a $\Rightarrow$ x < - a ; x > a	Examples: (1.) |x| < 6 $\Rightarrow$ -6 < x < 6 (2.) | x + 2| < 7 $\Rightarrow$ -7 < x+2 < +7 $\Rightarrow$ -9 < x < 5 (3.) |2x – 3 | > 9 $\Rightarrow$ 2x – 3 < - 9 $\Rightarrow$ 2x < - 6 $\Rightarrow$ x < -3 2x – 3 > 9 $\Rightarrow$ 2x > 12 $\Rightarrow$ x > 6
Quadratic Equations $ax^2 + bx + c = 0$ a,b and c are constants; a $\neq 0$ Can be solved by *factoring* or using the *quadratic formula* : $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ **Case1**: $b^2 - 4ac > 0$; Two *distinct* real roots. **Case2 :** $b^2 - 4ac = 0$; One double root x = -b/2a **Case3:** $b^2 - 4ac < 0$; No real roots.	Examples: **(1.)** $x^2 - 7x + 6 = 0$; by factoring:(x – 1)(x - 6)=0; x=1;x = 6 By Formula : a = 1 ; b = -7 ; c = 6 $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{49 - 4(1)(6)}}{2(1)}$ $x = \frac{7 \pm \sqrt{25}}{2}$; x = 1 ; x = 6 . **(2.)** $x^2 - 3x + 7 = 0$; a= 1 , b = -3 , c = 7; $b^2 - 4ac = (-3)^2 - 4(1)(7) = 9 - 28 = -19 < 0$ No real roots for this equation.

Higher Equations

BiQuadratic Equations

A. $ax^4 + bx^2 + c = 0$

a,b,c are constants ; $a \neq 0$

set $y = x^2$,the equation becomes

$ay^2 + by + c = 0$ which is a quadratic equation.

Note that $y = x^2 \geq 0$

The solution of the biquadratic equation is x2 instead of x.

B. Equations of degree ≥ 3

Substitute the divisors of the constant term in the equation:

f(a) = 0 , then x = a is a root of the equation and (x-a) is a factor of f(x),see Example2

$x^3 - 4x^2 - 9x + 36 = 0$

the divisors of 36: $\pm 1, \pm 2, \pm 3, \pm 4, \ldots$

x=- 1: $(-1)^3 - 4(-1)^2 - 9(-1) + 36 = 40$

-1 is not a root.

x= 1: $(1)^3 - 4(1)^2 - 9(1) + 36 = 24$

x= 1 is not a root.

x=4 : $(4)^3 - 4(4)^2 - 9(2) + 36 = 0$

x =4 is a root.

Similarly

f(-3) = 0 ; -3 is a root.

f(3)= 0 ; 3 is a root

Example1: $5x^4 - 3x^2 - 2 = 0$

a = 5 ,b = -3 , c = -2

$$x^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - 4(5)(-2)}}{2(5)}$$

$x^2 = \dfrac{3 \pm \sqrt{49}}{10}$; $x^2 = -4/10$ *No real solution*(remember $x^2 \geq 0$**)**

$X^2 = 1$; $x = \pm 1$

Recall : $x^2 = a$; a>0 then $x = \pm \sqrt{a}$

Remove the square, $\pm$square root the answer.

Example2: $x^3 - 4x^2 - 9x + 36 = 0$

You may solve this by factoring by taking them two at a time:

$x^3 - 4x^2 - 9x + 36 = 0$

$x^2(x - 4) - 9(x - 4) = 0$

$(x- 4)(x^2 - 9) = 0$

Either x -4 = 0 ; x = 4

Or $x^2 = 9$; $x = \pm \sqrt{9}$; $x = \pm 3$

Example3: $x^3 - 2x^2 - 5x + 6 = 0$

The divisors of 6 : $\pm 1, \pm 2, \pm 3, \pm 6$

$f(1) = (1)^3 - 2(1)^2 - 5(1) + 6 = 0$;

1 is a root .

$f(-2) = (-2)^3 - 2(-2)^2 - 5(-2) + 6 = 0$;

-2 is a root .

$f(3) = (3)^3 - 2(3)^2 - 5(3) + 6 = 0$;

3 is a root .

The roots are : 1, -2 and 3.

Square roots Equations

Equations involving square roots can be solved by squaring both sides.

$\sqrt{x-1} = 3$; note that **x -1**≥ 0

$\Rightarrow x \geq 1$

i.e valid solutions should be greater or equal to **1**.

Squaring both sides :

$x - 1 = 9 \Rightarrow x = 10$ accepted being ≥ 1

Example: $\sqrt{2x-1} = 2x - 3$

Valid solutions : $2x - 1 \geq 0$; $x \geq \dfrac{1}{2}$

Squaring both sides :

$2x - 1 = (2x - 3)^2$

$2x - 1 = 4x^2 - 12x + 9$

$4x^2 - 14x + 10 = 0$

Solving by factoring or by the quadratic formula :

x = 1 accepted being ≥ 0.5

$x = 10/4 = 2.5$ accepted being ≥ 0.5

Simultaneous Equations

Two equations with two unknowns.
$ax + by = c$
$dx + fy = e$
There are many ways to solve such equations.The easiest two are by substitution and by elimination.

By Substitution

Find x in terms of y in any of the equations and substitute it in the other equation.
Substitution is good when you have the coefficient of at least one of the unknowns is 1.
e.g.
$x + 7y = 10$;it is easy to set $x = 10 - 7y$;
while in $31x - 17y = 103$, you will face
$x = (103 + 17y)/31$,
in this case use elimination for less trouble with arithmetic.

By Elimination

Is the process where you get rid of one of the unknowns.
If it is possible to make the x has same coefficient in both equations, then by subtraction, you will be able to get rid of x.

We do so by multiplying each equation with the coefficient of x in the other equation(see Example2)

Example1: $x + 3y = 11$ ------(1)
$5x - 2y = 4$ -------(2)

From (1), $x = 11 - 3y$ substitute this in

(2) : $5(11 - 3y) - 2y = 4$

$\Rightarrow 55 - 15y - 2y = 4 \Rightarrow -17y = -51$
$y = 3$.

Now $x = 11 - 3y = 11 - 3(3) = 2$
$(x,y) = (2,3)$

Checking: substitute the values of x and y in any of the equations ,if these values satisfy the equation ,then your solution is correct:
$x + 3y = 11$
$2 + 3(3) = 11$.

Example 2 : $5x - 3y = -6$ ------(1)
$3x + 7y = 14$ ------(2)

Multiply (1) by 3 : $15x - 9y = -18$

Multiply (2) by 5 : $15x + 35y = 70$

Subtracting : $-9y - 35y = -18 - 70$
$-44y = -88$; **$y = 2$**

Now substitute this in any of the equations: $5x - 3y = -6$
$5x - 3(2) = -6$
$5x - 6 = -6$
$5x = 0$; $x = 0/5 = 0$

$(x,y) = (0, 2)$