

International Institute
for Technology and Management



Unit 05a : Mathematics 1

Handout #17

Partial Derivatives III

Study Guide pp 87-88 ;Anthony & Biggs pp 139-142

Topic	Interpretation
<p>Optimization: several variables Suppose a company produces two products X and Y.</p> <p>If P_x is the selling price of one unit of X, the total revenue obtained by producing x amount: xP_x</p> <p>Similarly, if P_y is the selling price of one unit of Y, the total revenue obtained by producing x amount: yP_y</p> <p>The total revenue obtained by producing x and y :</p> $TR(x,y) = xP_x + yP_y$ <p>If $TC(x,y)$ is the total cost ,the the Profit function:</p> $\pi = TR - TC = xP_x + yP_y - TC$ <p>Relation between Prices & Quantities: <u>Case 1:</u> P_x & P_y are constants: Perfect competition or efficient Small firm.(Example 1)</p>	<p><u>Example 1:</u> A small firm manufacturers two goods X and Y and the Market price of these goods is unaffected by the by the level of the firm's production. The firm's cost function is $C(x,y) = 2x^2 + 2y^2 + xy$ and the market price of X is \$12 per unit and the market price of Y is \$18 per unit. <i>Write the firm's profit function as an expression in terms of x and y:</i> $TR = xP_x + yP_y = 12x + 18y$ $\pi = TR - TC = 12x + 18y - 2x^2 - 2y^2 - xy$ <i>- Determine the number of units of each that the firm should produce to maximise its profit:</i>A usual maximization problem: Let $\pi = f(x,y)$; $f_{11} = 12 - 4x - y = 0$; $f_{22} = 18 - 4y - x = 0 \Rightarrow x = 2 , y = 4$ $f_{11} = -4 , f_{22} = -4 ; f_{12} = 1$ $\Rightarrow (f_{11})(f_{22}) - f_{12}^2 = 15 > 0$ Since $f_{11} = -4 < 0 \Rightarrow (2,4)$ maximizes f. Maximum Profit: $f(2,4) = 12(2) + 18(4) - 2(2)^2 - 2(2)^2 - (2)(4) = 76$</p> <p><u>Example 2:</u> A firm is monopoly has total cost function: $C(x,y) = 5 + x^2 + y^2 - xy$ The inverse demand functions for X & Y : $P_x = 5 - 0.5x ; P_y = 24 - y$ <i>The profit function :</i> $TR = xP_x + yP_y = x(5 - 0.5x) + y(24 - y)$ $TR = 5x - 0.5x^2 + 24y - y^2$</p>

<p><u>Case 2:</u> P_x depends only on x , P_y depends only on y: Monopoly ,No interaction</p> <p>between the markets for X & Y P_x & P_y are inverse demand functions.(Example 2)</p> <p><u>Case 3:</u> each P_x & P_y depends on both x and y : amount of X on the market affects the demand of Y and vice versa. Equivalently the price of X affects the demand for X as well as the Demand for Y and vice versa.(Example 3)</p>	$\pi = TR - TC$ $= 5x - 0.5x^2 + 24y - y^2 - 5 - x^2 - y^2 + xy$ $= 5x + 24y - 1.5x^2 - 2y^2 + xy - 5$ <p><u>Example 3:</u> a firm is the only producer of X and Y ; the demand for X is given by : $x = 2 - 2P_x + P_y$ the demand for Y is given by : $y = 13 + P_x - 2P_y$</p> <p><i>find the profit function</i> knowing that the total cost function: $C(x,y)=5+x^2 +y^2 - xy$</p> <p>$TR = xP_x + yP_y$; to get P_x and P_y Rearrange the equations $x = 2 - 2P_x + P_y \Rightarrow - 2P_x + P_y = x - 2$ $y = 13 + P_x - 2P_y \Rightarrow P_x - 2P_y = y - 13$</p> <p>Solve for P_x & P_y Simultaneously : $P_x = (17 - 2x - y)/3$ $P_y = (28 - x - 2y)/3$ $TR = xP_x + yP_y$ $= (x/3)(17 - 2x - y) + (y/3)(28 - x - 2y)$</p> $\pi = TR - TC$ $= -5 + (17/3)x + (28/3)y - (5/3)x^2$ $- (5/3)y^2 + (1/3)xy$
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