International Institute
for Technology and Management

Unit 05a : Mathematics 1 **Handout #17**

Partial Derivatives III

Study Guide pp 87-88 ;Anthony & Biggs pp 139-142

Topic	Interpretation
Optimization: several variables Suppose a company produces two products X and Y. If P_x is the selling price of one unit of X,the total revenue obtained by producing x amount: xP_x Similarly, if P_y is the selling price of one unit of Y,the total revenue obtained by producing x amount: yP_y The total revenue obtained by producing x and y : $TR(x,y) = xP_x + yP_y$ If TC(x,y) is the total cost ,the the Profit function: $\pi = TR - TC = xP_x + yP_y - TC$ **Relation between Prices & Quantities:** Case 1: P_x & P_y are constants: Perfect competition or efficient Small firm.(Example 1)	Example1: A small firm manufacturers two goods X and Y and the Market price of these goods is unaffected by the by the level of the firm's production.The firm's cost function is $C(x,y) = 2x^2 + 2y^2 + xy$ and the market price of X is \$12 per unit and the market price of Y is \$18 per unit. *Write the firm's profit function* as an expression in terms of x and y: $TR = xP_x + yP_y = 12x + 18y$ $\pi = TR - TC = 12x + 18y - 2x^2 - 2y^2 - xy$ - *Determine the number of units* of each that the firm should produce to *maximise its profit*:A usual maximization problem: Let $\pi = f(x,y)$;$f_{11} = 12 - 4x - y = 0$; $f_{22} = 18 - 4y - x = 0 \Rightarrow x = 2, y = 4$ $f_{11} = -4$, $f_{22} = -4$; $f_{12} = 1$ $\Rightarrow (f_{11})(f_{22}) - f_{12}^2 = 15 > 0$ Since $f_{11} = -4 < 0 \Rightarrow (2,4)$ maximizes f. Maximum Profit: f(2,4)= $12(2)+18(4)-2(2)^2-2(2)^2-(2)(2) = 76$ Example 2: A firm is monopoly has total cost function: $C(x,y) = 5+x^2 + y^2 - xy$ The inverse demand functions for X & Y : $P_x = 5 - 0.5x$; $P_y = 24 - y$ *The profit function* : $TR = xP_x + yP_y = x(5-0.5x) + y(24-y)$ $TR = 5x - 0.5x^2 + 24y - y^2$

Case 2: P_x depends only on x , P_y depends only on y:
Monopoly ,No interaction

between the markets for X & Y
P_x & P_y are inverse demand functions.(Example 2)

Case 3: each P_x & P_y depends on both x and y : amount of X on the market affects the demand of Y and vice versa. Equivalently the price of X affects the demand for X as well as the Demand for Y and vice versa.(Example 3)

π = TR − TC
$=5x - 0.5x^2 + 24y - y^2 - 5 - x^2 - y^2 + xy$
$= 5x + 24y - 1.5x^2 - 2y^2 + xy - 5$

Example 3: a firm is the only producer of X and Y ; the demand for X is given by :
$x = 2 - 2P_x + P_y$
the demand for Y is given by :
$y = 13 + P_x - 2P_y$

find the profit function knowing that the total cost function: $C(x,y)=5+x^2 + y^2 - xy$

TR = $xP_x + yP_y$; to get P_x and P_y
Rearrange the equations
$x = 2 - 2P_x + P_y \Rightarrow -2P_x + P_y = x - 2$
$y = 13 + P_x - 2P_y \Rightarrow \quad P_x - 2P_y = y - 13$

Solve for P_x & P_y Simultaneously :
$P_x = (17 - 2x - y)/3$
$P_y = (28 - x - 2y)/3$
TR = $xP_x + yP_y$
$=(x/3)(17 - 2x - y)+(y/3)(28 - x - 2y)$

π = TR − TC
$=-5 + (17/3)x + (28/3)y - (5/3)x^2$
$-(5/3)y^2 + (1/3)xy$