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International Institute
for Technology and Management

Partial Derivatives I

Study Guide pp 79-86 ;Anthony & Biggs pp 113-115,119

Topic	Interpretation
Functions of several variables f: $\Re^2 \to \Re$ is a function f of **two** variables which assigns to each (x,y) in $\Re^2$ a unique value f(x,y) in $\Re$. The extension to a function of **n** variables is called a function of several variables. **Notations:** Z = f(x,y) Example: $Z = f(x,y) = x^2 - 3y$ $Z = f(x,y) = x + e^{x^2+y^2}$ $Z = f(x,y) = x^2y + 7xy + y^3$ Calculate f(0 , 1) for the above functions.	A function f may be thought of as a *machine* which accepts an input x and produces f(x).The input for a function of two variables is the pair (x,y). f(0,1) : Replace x by 0 and y by 1 $Z = f(x,y) = x^2 - 3y \Rightarrow f(0,1) = -3$ $Z = f(x,y) = x + e^{x^2+y^2} \Rightarrow f(0,1) = e$ $Z = f(x,y) = x^2y + 7xy + y^3 \Rightarrow f(0,1) = 1$
Partial Derivatives Let Z = f(x,y) be a function of two variables.The *partial derivative* of f with respect to **x** is obtained by treating **y** as a *constant* and taking the derivative with respect to **x** . **Notations :** $\frac{\partial f}{\partial x}$, f_x , $\frac{\partial Z}{\partial x}$, Z_x , f_1 Example: $Z = f(x,y) = x^2y + 7xy + y^2$ Find the partial derivatives of f i.e. find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$	The *partial derivative* of f with respect to **y** is obtained by treating **x** as a *constant* and taking the derivative with respect to **y** . **Notations :** $\frac{\partial f}{\partial y}$, f_y , $\frac{\partial Z}{\partial y}$, Z_y , f_2 $Z = f(x,y) = x^2y + 7xy + y^2$ $f_1 = \frac{\partial f}{\partial x} = 2xy + 7y$ $f_2 = \frac{\partial f}{\partial y} = x^2 + 7x + 2y$

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2^{nd} Order Partial Derivatives

Are the partial derivatives with respect to x and y of f_1 and f_2

Example1:

$Z = f(x,y) = x^2y + 7xy + y^2$

$f_1 = \frac{\partial f}{\partial x} = 2xy + 7y$

has **two** partial derivatives one with respect to x and the other with respect to y .

Similarly $f_2 = \frac{\partial f}{\partial y} = x^2 + 7x + 2y$

has **two** partial derivatives one with respect to x and the other with respect to y .

Hence **there are four second partial derivatives of f .**

Notations:

$Z = f(x,y)$

Partial derivatives :

A. $\frac{\partial f}{\partial x}$, f_x , $\frac{\partial Z}{\partial x}$, Z_x , f_1

2^{nd} order Partial Derivatives:

w.r.t. x :

1. $\frac{\partial^2 f}{\partial x^2}$, f_{xx} , $\frac{\partial^2 Z}{\partial x^2}$, Z_{xx} , f_{11}

w.r.t. **y :**

2. $\frac{\partial^2 f}{\partial y \partial x}$, f_{xy} , $\frac{\partial^2 Z}{\partial y \partial x}$, Z_{xy} , f_{12}

B. $\frac{\partial f}{\partial y}$, f_y , $\frac{\partial Z}{\partial y}$, Z_y , f_2

2^{nd} order Partial Derivatives:

w.r.t. x :

3. $\frac{\partial f}{\partial x \partial y}$, f_{yx} , $\frac{\partial^2 Z}{\partial x \partial y}$, Z_{yx} , f_{21}

Example1:

$Z = f(x,y) = x^2y + 7xy + y^2$

$f_1 = \frac{\partial f}{\partial x} = 2xy + 7y$

$f_2 = \frac{\partial f}{\partial y} = x^2 + 7x + 2y$

For the 2^{nd} order partial derivatives :

Differentiate f_1 w.r.t **x** to get f_{11}

$f_{11} = 2y = \frac{\partial^2 f}{\partial x^2}$

Differentiate f_1 w.r.t **y** to get f_{12}

$f_{12} = 2x + 7 = \frac{\partial^2 f}{\partial y \partial x}$

Differentiate f_2 w.r.t **x** to get f_{21}

$f_{21} = 2x + 7 = \frac{\partial f}{\partial x \partial y}$

Differentiate f_2 w.r.t **y** to get f_{22}

$f_{22} = 2 = \frac{\partial^2 f}{\partial y^2}$

Note that $f_{12} = f_{21}$

w.r.t. **y :**

4. $\frac{\partial^2 f}{\partial y^2}$, f_{yy} , $\frac{\partial^2 Z}{\partial y^2}$, Z_{yy} , f_{22}

This function is called a "well behaved function" such as those which are presumed to occur in economic models.

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