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International Institute
for Technology and Management

Unit 05a : Mathematics 1 **Handout #1**

Basics I: Arithmetic

Topic	Interpretation
The Real Numbers N:Natural (counting) Numbers W:Whole Numbers Z:Integers Q:Rational Numbers I:Irrational numbers $N \subseteq Z \subseteq Q$ $R = Q \cup I$	 1,2,3,4,……. 0,1,2,3,4,….. …..,-3,-2,-1,0,1,2,3,…… All numbers of the form a/b ,where a and b are integers with $b \neq 0$ e.g. 3/2 , - 2/7 , 5 = 5/1 Real numbers that are not Rational e.g. π All the above numbers are *real* numbers.
Order of operations Follow the following order as they occur working from left to right: 1.Parenthses & square brackets 2.Powers 3.Multiplication or division 4.Addition or subtraction	$\frac{2(-2-5)^2+4(5)}{-3+4} = \frac{2(-7)^2+20}{1}$ $= \frac{2(49)^2+20}{1}$ =2(49)+20 =98+20= 118 $\frac{-11-(-12)-4\times 5}{4(-2)-(-6)(-5)} = \frac{-11+12-20}{-8-(+30)} = \frac{-19}{-38}$ $= \frac{19}{38} = \frac{1}{2}$ = 0.5
Absolute Value $\lvert x\rvert = \begin{cases} -x & \text{if } x<0 \\ +x & \text{if } x \geq 0 \end{cases}$ $\lvert x \rvert \geq 0$ for every x $\lvert x-y \rvert \neq \lvert x\rvert - \lvert y\rvert$ $\lvert x+y \rvert = \lvert x\rvert + \lvert y\rvert$ for x,y having the same sign.	| -5 | = 5 ; | 5 | = 5 |3 – 7 | = | -4 | = 4 |3| - |7| = 3 – 7 = - 4 |5+4| = |9| = 9 |5| + |4| = 5 + 4 = 9
Fractions Addition $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$	$\frac{3}{7} - \frac{5}{8} = \frac{8\times 3 - 7\times 5}{56} = \frac{24-35}{56} = \frac{-11}{56}$

Square roots There are two numbers whose square is 25 : -5 and 5 $(-5)^2$ = -5 x -5 = 25 5^2 = 5 x 5 = 25 The *positive* one, 5, is called the square root of 25. $\sqrt{x} \geq 0$ for every real number x For $\sqrt{x}$ to exist , $x \geq 0$ $(\sqrt{x})^2 = \sqrt{x} \times \sqrt{x} = x$ $\sqrt{x+y} \neq \sqrt{x} + \sqrt{y}$ $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$ $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$	$\sqrt{49} = 7$ since 7^2 = 49 $\sqrt{-25}$ does not exist. $\sqrt{16+9} = \sqrt{25} = 5$ $\sqrt{16} + \sqrt{9} = 4 + 3 = 7$ $4\sqrt{5} \times 7\sqrt{3} = 28\sqrt{15}$ $\frac{\sqrt{2}}{\sqrt{3}} = \sqrt{\frac{2}{3}}$ $\sqrt{2} \times \sqrt{2} = 2$ $7\sqrt{2} \times 3\sqrt{2} = 21 \times 2 = 42$ $\frac{5}{\sqrt{3}} = \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$ Estimate $\sqrt{73}$ since 8^2 = 64 and 9^2 =81 $\sqrt{73}$ must be between 8 and 9.
Exponents $x^m \times x^n = x^{m+n}$ $\frac{x^m}{x^n} = x^{m-n}$ $(x^m)^n = x^{mn}$ $\frac{1}{x^m} = x^{-m}$ $\sqrt{x} = x^{\frac{1}{2}}$ $\sqrt[n]{x^m} = x^{\frac{m}{n}}$	$2x^3 \times 5x^4 = 10x^7$ $\frac{-12x^7}{6x^5} = -2x^2$ $(2x^2)^3 = 2^3 x^6 = 8x^6$ $\frac{1}{x^3} = x^{-3}$ $\sqrt{25x^4} = 5x^{\frac{4}{2}} = 5x^2$ $\sqrt[5]{x^3} = x^{\frac{3}{5}}$
Polynomials In the expression $2x^3$, x is called a *variable* because it can assume any number of different values. 2 is called the coefficient. The highest power that appears in a polynomial is the degree of the polynomial. $2x - 5x^3 + 7x^2$ is of degree 3	Like terms of a polynomial can be added or subtracted, unlike terms can not. $9x^5 - 15x^5 = -6x^5$ $3x + 4x^2 = 3x + 4x^2$ Multiplication : (2x − 1)(3x + 5) = (2x)(3x) + (2x)(5) −(1)(3x) −(1)(5) = $6x^2$+ 10x -3x -5 = $6x^2$ +7x -5

Factoring

The number 10 can be written as 5x2, 1x10,….
The numbers in each product are called *factors*, the process of writing 10 as a product of factors is called *factoring*.

Difference of two squares
$(a-b)(a+b) = a^2 - b^2$

$a^2 + b^2$ can not be factored in real numbers and is always positive; $x^2 + 5 > 0$ for every x real number.

Difference of two cubes
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Sum of two cubes
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

Perfect Squares
$(a+b)^2 = (a+b)(a+b)$
$= a^2 + 2ab + b^2$
$(a-b)^2 = (a-b)(a-b)$
$= a^2 - 2ab + b^2$

Perfect Cubes
$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Factoring Trinomial
A trinomial is a polynomial with three terms.Our concern here is those of degree 2 :
$x^2 + bx + c$;when it is possible,can be factored into two factors $(x + m)(x+n)$
Look for the factors of the constant term *c* whose sum (or difference) is *b* ; being factors of c ,their product is c .
$x^2 + 5x + 4$
the factors of 4 whose sum is 5 are 1 and 4
$x^2 + 5x + 4 = (x+1)(x+4)$

Example1: factor $12x - 18y$
Both 12x and 18y are divisible by 6 :
$6(2x) - 6(3y) = 6(2x - 3y)$

Example2: $8x^3 - 9x^2 + 15x$
Each of these terms is divisible by x :
$x(8x^2) + x(-9x) + x(15) = x(8x^2 - 9x + 15)$

Example3: $5(4x-3)^3 - 2(4x-3)^2$
$(4x-3)^2$ is the common factor :
$(4x-3)^2 [\ 5(4x-3) - 2] = (4x-3)^2(20x-17)$

Example4: $x^2 - 16 = x^2 - 4^2 = (x-4)(x+4)$

Example5: $81x^4 - 16 = (9x^2)^2 - (2^2)^2$
$= (9x^2 - 4)(9x^2+4)$
$= (3x-2)(3x+2)\ (9x^2+4)$

Example6: $x^3 - 8 = x^3 - 2^3$
$= (x-2)(x^2 + 2x + 4)$

Example7: $y^3 + 125 = y^3 + 5^3$
$= (y+5)(y^2 - 5y + 25)$

Example8: $(2x - 7)^2 = (2x)^2 - 2(2x)(7) + 7^2$
$= 4x^2 - 28x + 49$

Example9: $(q - 2)^3$
$= q^3 - 3(q^2)(2) + 3q(2^2) - 2^3$
$= q^3 - 6q^2 + 12q - 8$

Example9: Factor $x^2 - x - 6$
The factors of 6 whose sum or difference is – 1 are 2 and - 3 (notice their product is – 6) : $(x - 3)(x+2)$

Example10: Factor $4y^2 - 11y + 6$

Here the factors should look like:
(2y)(2y) or (4y)(y)
We do it by trial :we need 6 at the end ,i.e. (-1)(-6) ,(1)(6) ,(2)(3) or (-2)(-3)
$(2y - 1)(2y - 6) = 4y^2 - 14y + 6$ NO
$(2y - 2)(2y - 3) = 4y^2 - 14y + 6$ NO
$(4y - 3)(y - 2) = 4y^2 - 11y + 6$ YES