



February 16, 2009
GROUP: A

Unit: 05a – Mathematics 1
Duration: 90 minutes

Answer all of the following questions:

1. (a) Suppose that $f(x,y) = x^2y + y^2$.
Let $x = 3t^2 + 3$ and $y = t^3 - 7$, Use the chain rule to
find $F'(2)$.

(10 Marks)

- (b) If $x^{\sqrt{y}} = 7$ find $\frac{dy}{dx}$

(10 Marks)

2. (a) Find and classify the stationary points of the function

$$f(x,y) = x^2 - 2x - y^3 + y^2 + 8$$

(10 Marks)

- (b) The function $f(x,y)$ is defined for $x, y > 0$ by

$$f(x,y) = \frac{ye^{2y}}{x^a},$$

where a is a fixed real number. Find expressions for the partial
derivatives

$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}$$

Determine the values of a for which the function will satisfy the
equation

$$yx^2 \frac{\partial^2 f}{\partial x^2} - 3y \frac{\partial^2 f}{\partial y^2} + 12f = 0.$$

(15 Marks)

3. Use the Lagrange multiplier method to find the maximum value of

$$\left(\frac{1}{x^2} + \frac{1}{y^2} \right)^{-1/2}$$

among all positive x, y satisfying $x + y = \sqrt{2}$

(10 Marks)

4. If

$$f(x, y) = y \ln \left(\frac{y}{x} \right) + x e^{x/y}$$

(defined for positive x and y), find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ and show that

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = f.$$

(15 Marks)

5. (a) A firm is the only producer of two goods, X and Y . The demand equations for X and Y are given by

$$x = 50 - \frac{1}{2}p_X, \quad y = 240 - 2p_Y,$$

where x and y are the quantities of X and Y demanded (respectively) and p_X, p_Y are (respectively) the prices of X and Y . The firm's joint total cost function (that is, the cost of producing x of X and y of Y) is

$$x^2 + 2xy + y^2 + 10.$$

Find an expression in terms of x and y for the profit function. Determine the quantities x and y that maximise the profit.

(15 Marks)

(b) A consumer has utility function

$$u(x, y) = 3 \ln x_1 + \ln x_2$$

for two goods, X_1 and X_2 . (Here, x_1 and x_2 are, respectively, the amounts of X_1 and X_2 consumed.) Suppose that each unit of X_1 costs $\$p_1$ and each unit of X_2 costs $\$p_2$, and that the consumer has a budget of M to spend on these two goods. By using the Lagrange multiplier method, determine the quantities x_1^* and x_2^* of X_1 and X_2 that maximise the consumer's utility function subject to the constraint on his budget.

(15 Marks)

END OF PAPER