International Institute for Technology and Management

February 15th, 2005 **Unit : 05a – Mathematics 1**

Assignment 5

SOLUTION

1. Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of the following functions:

a. f(x,y) = $5x^{\frac{2}{3}}y^{\frac{1}{4}}$;f_1= (10/3) $x^{\frac{-1}{3}}y^{\frac{1}{4}}$; f_2= (5/4) $x^{\frac{2}{3}}y^{\frac{-3}{4}}$

b. f(x,y) = $x^2y+e^{xy^2}$; f_1=2xy + $y^2 e^{xy^2}$; f_2=x^2 + 2xy e^{xy^2}

c. f(x,y) = $y^{\sqrt{x}}$ = $e^{\sqrt{x}\ln y}$; f_1= $\frac{\ln y}{2\sqrt{x}}\, e^{\sqrt{x}\ln y}$; f_2= $\frac{\sqrt{x}}{y}\, e^{\sqrt{x}\ln y}$

d. f(x,y) =Ln(x^2 + y^2); f_1= $\frac{2x}{x^2+y^2}$; f_2 = $\frac{2y}{x^2+y^2}$

2. Consider the function: 3^{x^2y} **= 25. Find** $\frac{dy}{dx}$ **.**

$$\frac{dy}{dx} = -\frac{\partial g/\partial x}{\partial g/\partial y} = \frac{(2xy)3^{x^2y}\ln 3}{(x^2)3^{x^2y}\ln 3} = \frac{2y}{x}$$

3. For the function: f(x,y) = $(x^4+y^4)^{\frac{1}{2}}(x^2y+x^3)$

Find $\frac{\partial f}{\partial x}$ **and** $\frac{\partial f}{\partial y}$ **then Show that : x**$\frac{\partial f}{\partial x}$ **+ y**$\frac{\partial f}{\partial y}$ **= 5f**

$$\frac{\partial f}{\partial x} = \frac{1}{2}(4x^3)(x^4+y^4)^{\frac{-1}{2}}(x^2y+x^3)+(x^4+y^4)^{\frac{1}{2}}(2xy+3x^2)$$

$$x\frac{\partial f}{\partial x} = (2x^4)(x^4+y^4)^{\frac{-1}{2}}(x^2y+x^3)+x(x^4+y^4)^{\frac{1}{2}}(2xy+3x^2)$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}(4y^3)(x^4+y^4)^{\frac{-1}{2}}(x^2y+x^3)+(x^4+y^4)^{\frac{1}{2}}(x^2)$$

$$y\frac{\partial f}{\partial y} = (2y^4)(x^4+y^4)^{\frac{-1}{2}}(x^2y+x^3)+y(x^4+y^4)^{\frac{1}{2}}(x^2)$$

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} =$$

$$(x^4+y^4)^{\frac{-1}{2}}(x^2y+x^3)(2x^4+2y^4)+(x^4+y^4)^{\frac{1}{2}}(2x^2y+3x^3+yx^2)$$

$$2(x^4+y^4)^1(x^4+y^4)^{\frac{-1}{2}}(x^2y+x^3)+(x^4+y^4)^{\frac{1}{2}}(3x^2y+3x^3)$$

$$2(x^4+y^4)^{\frac{1}{2}}(x^2y+x^3)+3(x^4+y^4)^{\frac{1}{2}}(x^2y+x^3)$$

$$5(x^4+y^4)^{\frac{1}{2}}(x^2y+x^3) = 5f$$

4. Show that the function

$$f(x,y) = -1.5x^2 + 5x - 2y^2 + 24y + xy - 5$$

has a critical point at (4,7) and verify that it is a maximum

$f_1 = -3x + 5 + y = 0$; $f_2 = -4y + 24 + x = 0$
Solving simultaneously: x = 4 ; y = 7

$f_{11} = -3$, $f_{22} = -4$; $f_{12} = 1 \Rightarrow (f_{11})(f_{22}) - f_{12}^{\,2} = 11 > 0$
Since $f_{11} = -3 < 0 \Rightarrow$ (4,7) maximizes f .

5. A small firm manufacturers two goods X and Y and the Market price of these goods is unaffected by the by the level of the firm's production.The firm's cost function is $C(x,y) = 2x^2 + 2y^2 + xy$ and the market price of X is \$12 per unit and the market price of Y is \$18 per unit.

a. Write the firm's profit function as an expression in terms of x and y .
Total Revenue = $xP_x + yP_y = 12x + 18y$
Profit = Total Revenue – Total Cost
Profit = $f(x,y) = 12x + 18y - 2x^2 - 2y^2 - xy$

b. Determine the number of units of each that the firm should produce to maximise its profit.

$f_1 = 12 - 4x - y = 0$; $f_2 = 18 - 4y - x = 0$
Solving simultaneously: x = 2 ; y = 4

$f_{11} = -4$, $f_{22} = -4$; $f_{12} = 1 \Rightarrow (f_{11})(f_{22}) - f_{12}^{\,2} = 15 > 0$
Since $f_{11} = -4 < 0 \Rightarrow$ (2,4) maximizes f.

c. Calculate the maximum profit:
Max Profit = f(2,4) = $12(2)+18(4)-2(2)^2-2(2)^2-(2)(2)$
= 76

End of Answers