International Institute for
Technology and Management
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Unit 05a: Mathematics 1 – (MathA) **Assignment – 2**

1. Determine the derivatives of the following functions :

a. y = **(3lnx – x)(lnx – 2)^2** of the form u x v

u = 3lnx – x $\Rightarrow$ u' = 3/x - 1
v = (lnx-2)^2 $\Rightarrow$ v' = 2(lnx – 2)(1/x)
y' = u'v + v'u = (3/x - 1) (lnx-2)^2 + 2(lnx – 2)(1/x)(3lnx – x)

b. y = $\left(\sqrt{1+x^2}\right)e^x$ of the form u x v

u = $\sqrt{1+x^2}$ $\Rightarrow$ u' = $\dfrac{2x}{2\sqrt{1+x^2}} = \dfrac{x}{\sqrt{1+x^2}}$

v = e^x $\Rightarrow$ v' = e^x

y' = u'v + v'u = ($\dfrac{x}{\sqrt{1+x^2}}$)e^x + (e^x)($\sqrt{1+x^2}$)

c. y = $\dfrac{x^2-1}{x} + e^{\frac{-x^2}{2}} = x - \dfrac{1}{x} + e^{\frac{-x^2}{2}}$

y' = 1 – (-1/x^2) + (-2x/2) $e^{\frac{-x^2}{2}}$ = 1 + 1/x^2 - x$e^{\frac{-x^2}{2}}$

d. y = $\sqrt{\dfrac{1-x}{1+x}}$ of the form $\sqrt{U}$, its derivative is $\dfrac{U'}{2\sqrt{U}}$

$$U = \frac{1-x}{1+x} \Rightarrow U' = \frac{-2}{(1+x)^2} \text{ , y' = } \frac{1}{2\sqrt{\frac{1-x}{1+x}}} \times \frac{-2}{(1+x)^2}$$

e. y = $\dfrac{1-\sin x}{1+\sin x}$ of the form u/v

u = 1 – sinx $\Rightarrow$ u' = - cosx , v = 1 + sinx $\Rightarrow$ v' = cosx

$$y' = \frac{u'v - v'u}{v^2} = \frac{-\cos x(1+\sin x) - \cos x(1-\sin x)}{(1+\sin x)^2}$$

$$y' = \frac{-\cos x - \cos x \sin x - \cos x + \cos x \sin x}{(1+\sin x)^2} = \frac{-2\cos x}{(1+\sin x)^2}$$

f. y = $\ln(\sqrt{x+\sqrt{x}})$ of the form ln U , its derivative (1/U)U'

U = $\sqrt{x+\sqrt{x}}$, U' = $\frac{1}{2\sqrt{x+\sqrt{x}}}\times\left(1+\frac{1}{2\sqrt{x}}\right)$

y' = $\frac{1}{\sqrt{x+\sqrt{x}}}\times\frac{1}{2\sqrt{x+\sqrt{x}}}\times(1+\frac{1}{2\sqrt{x}})$

= $\frac{1}{2(x+\sqrt{x})}\times(1+\frac{1}{2\sqrt{x}})$

(18 Marks)

2. Find all stationary points of the following functions and determine whether they are maxima,minima or inflection points: **(12 Marks)**

a. y = $x^4 - \frac{5}{3}x^3 + \frac{1}{2}x^2$

y' = $4x^3 - 5x^2 + x = 0 \Rightarrow x(4x^2 - 5x + 1) = 0$

x = 0 , x = 1 , x = ¼

x = 0 $\Rightarrow$ y = 0 , 1st point : (0,0)

x = 1 $\Rightarrow$ y = -1/6 , 2nd point : (1,-1/6)

x = ¼ $\Rightarrow$ y = 31/768 , 3rd point (1/4 , 31/768)

y'' = $12x^2 - 10x + 1$

<u>at (0,0)</u> : y''(0) = 1 > 0 $\Rightarrow$ x = 0 minimizes the function and the value of the minimum is 0 .

<u>at (1,-1/6)</u> : y''(1) = 3 > 0 $\Rightarrow$ x = 1 minimizes the function and the value of the minimum is -1/6 .

<u>at (1/4 , 31/768)</u> : y''(1/4) = $12(1/4)^2 - 10(1/4) + 1$

y''(1/4) = 12/16 – 10/4 + 1 = 12 – 40 + 16/16 = -12/16 < 0

$\Rightarrow$ x = ¼ maximizes the function and the value of the Maximum is 31/768

b. y = $\frac{x^2-x+1}{x} \Rightarrow$ y' = $\frac{x^2-1}{x^2} = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

x = 1 $\Rightarrow$ y = 1 , first point (1,1)

x = - 1 $\Rightarrow$ y = - 1 , second point (-1,-1)

y'' = $\frac{2}{x^3}$

<u>at (1,1)</u> : y''(1) = 2 > 0 $\Rightarrow$ x = 1 minimizes the function and the value of the minimum is 1 .

<u>at (-1,-1)</u> : y''(1) = - 2 < 0 $\Rightarrow$ x = - 1 maximizes the function and the value of the maximum is -1 .

c. y = x^2 **lnx , note that lnx is defined only for x>0**

y' = 2xlnx + x^2(1/x) = 2xlnx + x = x(2lnx + 1) = 0

Either x = 0 (rejected since x >0) or 2lnx + 1 = 0

lnx = - ½ $\Rightarrow$ x = $e^{-1/2}$

(Remember : lnx = a $\Rightarrow$ x = e^a)

$\Rightarrow$ y = $(e^{-1/2})^2 \ln e^{-1/2}$ = (e^{-1})(- ½lne) = -1/2e, **(lne = 1)**

the point is ($e^{-1/2}$, -1/2e)

y'' = (1)(2lnx+1) + x(2/x) = 2lnx + 3

at ($e^{-1/2}$, -1/2e) : y''($e^{-1/2}$) = 2ln $e^{-1/2}$ + 3 = 2(-1/2) + 3

= 2 > 0

$\Rightarrow$ x = $e^{-1/2}$ minimizes the function and the value of the minimum is -1/2e .

3. Find all stationary points of the following function and Specify their nature: **(8 Marks)**

$$f(x) = \frac{1}{12}x - \sqrt[3]{x} = \frac{1}{12}x - x^{\frac{1}{3}}$$

f'(x) = 1/12 – (1/3) $x^{-2/3}$ = 0 $\Rightarrow$ 1/12 = $1/3x^{2/3}$

$\Rightarrow$ $3x^{2/3}$ = 12 $\Rightarrow$ $x^{2/3}$ = 4 cubing both sides

$(x^{2/3})^3 = 4^3$ $\Rightarrow$ x^2 = 64 $\Rightarrow$ x = $\pm$ 8

x = 8 $\Rightarrow$ y = (1/12)(8) - $\sqrt[3]{8}$ = 8/12 – 2 = 2/3 – 2 = -4/3

1st point (8 , -4/3)

x = -8 $\Rightarrow$ y = (1/12)(8) - $\sqrt[3]{-8}$ = 8/12 – (-2)= 2/3 + 2

= -4/3

2nd point (8 , 8/3)

y'' = (-2/3)(-1/3)$x^{-5/3}$ = $\dfrac{2}{9\sqrt[3]{x^5}}$

<u>At (8 , -4/3)</u> : y''(8) = $\dfrac{2}{9\sqrt[3]{8^5}}$ > 0

$\Rightarrow$ x = 8 minimizes the function and the value of the minimum is -4/3 .

<u>At (- 8 , 8/3)</u> : y''(8) = $\dfrac{2}{9\sqrt[3]{(-8)^5}}$ < 0

$\Rightarrow$ x =- 8 maximizes the function and the value of the maximum is 8/3 .

4. A firm has average variable cost **(6 Marks)**

$$q + 5e^{2q^2-1} + \frac{\ln(2q^2-1)}{q}$$

and fixed costs of 8 .Find the total cost function and the marginal cost function.

AVC = $q + 5e^{2q^2-1} + \frac{\ln(2q^2-1)}{q}$ $\Rightarrow$ VC = q x AVC

VC = $q^2 + 5qe^{2q^2-1} + \ln(2q^2-1)$

TC = VC + FC = $q^2 + 5qe^{2q^2-1} + \ln(2q^2-1) + 8$

MC = $\frac{dTC}{dq}$ = TC' = 2q + 5($e^{2q^2-1} + q(4q)\, e^{2q^2-1}$) + $\frac{4q}{2q^2-1}$

MC = 2q + 5(1 + 4q^2) e^{2q^2-1} + $\frac{4q}{2q^2-1}$

5. Find the values of the constant $a \neq 0$ for the which the function

$$f(x) = \ln x - \frac{2}{a}x^2$$

admits a maximum.

f'(x) = 1 / x – 2(2 / a)x = 1 / x – 4x / a = 0

1 / x = 4x / a $\Rightarrow$ 4x^2 = a $\Rightarrow$ x^2 = a/4 $\Rightarrow$ x = $\pm\frac{\sqrt{a}}{2}$

f''(x) = -1 / x^2 – 4 / a

for f to admit a maximum, f''(x) < 0

at x = $\pm\frac{\sqrt{a}}{2}$, f''(x) = - 1/(a/4) – 4/a = - 8/a < 0

we need to have a > 0

(6 Marks)

END of QUESTIONS