International Institute for
Technology and Management

October 24 ,2009 **Duration : 1.5 hrs**

Unit 05a: Mathematics 1 – (Math A&B) Assignment – 1

1. Solve the following equations in the set of real numbers :

a. $\frac{2}{3}q^2 + 5q - \frac{17}{3} = 0$

b. $\begin{cases} 16x + 8y + 12 = 0 \\ 10 + 8x + 20y = 0 \end{cases}$

c. $\begin{cases} 3y - 3x^2 = 0 \\ 3x + 3y^2 = 0 \end{cases}$

d. $\left(4x^2 - 6x\right)e^{x^2} + 2e^{x^2} = 0$

e. $(\ln x)^2 + 2 \ln x - 1 = 0$

f. $\frac{2}{\sqrt{q}} - 2q = 0$

(18 Marks)

2. The demand equation of a good is given by $p = q^2 + 4q + 20$, if the supply equation is $p = -q^2 - 10q + 176$, determine the equilibrium price and quantity. **(6 Marks)**

3. Show that the graphs of the functions $f(x) = x^2 - 2x - 4$ and $g(x) = x - 8$ do not intersect, and sketch both graphs on the same diagram.Determine the positive values of the constant a such that graph of the function $h(x) = ax - 8$ *does* intersect the graph of f(x).

(16 Marks)

4. A firm is a monopoly for the good it produces.Its average variable cost function is $q^2 + 4$, where q is the quantity it produces, and it has fixed costs of 20.The demand function for its goods is given by $p + q = 20$, where p is the price. Find expressions in terms of q ,for the total revenue and the profit.Determine the production level q that gives maximum profit. What is the maximum profit.

(12 Marks)

5. Functions f and g are as follows:

$f(x) = x^4 + 2x^3 + 2x^2 + 2, \quad g(x) = -x^4 + 2x^3 + 18x^2 + 20.$

Show that the curves y = f(x) and y = g(x) intersect for exactly two values of x. Find these values of x. (Do not attempt to sketch the curves.)

(8 Marks)

6.

(Profit Functions) A company has a profit function given by

$$\pi(x) = 52x - x^2 - 276$$

where x denotes the quantity produced.

(a) Complete the square of the function $\pi(x)$.

(b) Find the x-intercepts and y-intercepts of $\pi(x)$.

(c) Which value of x gives the highest profit, and what is the amount of this maximum profit?

(d) Use the above information to sketch the graph of $\pi(x)$.

(e) Given that the company has a linear cost function, and that it costs \$532 for four units and \$564 for eight units, determine the cost $C(x)$ to produce x units.

(f) What is the revenue function $R(x)$ for this company?

(24 Marks)

7. Firm X can produce up to 30 items of product A. The profit derived from the manufacturing of x items of product A is given by the profit function

$\pi(x) = 45x - 350 - x^2$.

(a) Sketch the graph of the profit function.

(b) Find the break-even point.

(c) Find the value of x for which the profit is maximal, and give the maximal profit level.

(16 Marks)

END of QUESTIONS