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UNIVERSITY OF LONDON

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996 D05a ZA

BSc degrees in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Mathematics 1 (half unit)

Tuesday, 13 May 2003 : 10.00am to 12.00noon

Candidates should answer **NINE** of the following **ELEVEN** questions: **SEVEN** from Section A (60 marks in total) and **TWO** from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

Graph paper is provided. If used, it must be securely fastened inside the answer book.

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UL03/407

SECTION A

Answer all **SEVEN** questions from this section (60 marks in total)

1 The supply equation for a good is $q = p^2 + 7p - 2$ and the demand equation is $q = -p^2 - p + 40$, where p is the price. Sketch the supply and demand functions for $p \geq 0$. Determine the equilibrium price and quantity.

2 Suppose that c is some fixed positive number and let f be the function $f(x) = cx - \ln x$ (defined for $x > 0$). Find the minimum value of $f(x)$ for $x > 0$. Use your result to show that for all positive numbers x,

$$\ln x \leq cx + \ln(1/c) - 1.$$

3 Determine the integral $\displaystyle\int \frac{1}{x(\ln x)^2}\,dx$.
[Hint: You might find it useful to use a substitution.]

4 Find the values of x and y that minimise the function

$$f(x, y) = 10x^2 - 2xy + 10x + 5y^2 + 6y + 6$$

and verify that these values do indeed give a minimum.

5 A firm has production function $q(k, l) = 4k^{1/2}l^{1/6}$, where k represents capital and l represents labour. Capital costs \$3 per unit and labour costs \$1 per unit. Use the Lagrange multiplier method to determine the values of capital and labour that minimise the cost of producing an amount Q. What is this minimum cost, in terms of Q?

6 Express the following system of equations in matrix form, and solve it using a matrix method.

$$\begin{aligned} x + y + 3z &= 6 \\ 2x + y + z &= 1 \\ -5x - 2y + 2z &= 7. \end{aligned}$$

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7 Jane has opened a savings account with a bank, and they pay a fixed interest rate of 5% per annum, with the interest paid once a year, at the end of the year. She opened the savings account with a payment of \$100 on 1 January 2003, and will be making deposits of \$200 yearly, on the same date. What will her savings be after she has made N of these additional deposits? (Your answer will be an expression involving N.)

SECTION B

Answer **TWO** questions from this section (20 marks each)

8 (a) A firm is a monopoly for the good it produces. Its marginal cost function is $MC = 3q^2 + 4$ and it has fixed costs of 10. The demand equation for its good is given by $p + q = 20$, where p is the price. Find expressions, in terms of q, for the total revenue and profit. Determine the production level q that gives maximum profit, and find the maximum profit.

(b) A firm is the only producer of two goods, X and Y. The demand equations for X and Y are given by

$$x = 25 - \frac{1}{2}p_X, \quad y = 30 - p_Y,$$

where x and y are the quantities of X and Y demanded (respectively) and p_X, p_Y are (respectively) the prices of X and Y. The firm's joint total cost function (that is, the cost of producing x of X and y of Y) is

$$x^2 + 2xy + y^2 + 20.$$

Find an expression in terms of x and y for the profit function. Determine the quantities x and y that maximise the profit, and find the corresponding prices p_X, p_Y.

9 (a) Determine the integrals $\displaystyle\int \frac{x+1}{x^2+2x+8}\,dx$ and $\displaystyle\int \frac{x+3}{x^2+5x+4}\,dx$.

(b) The manufacturer of a new product anticipates that the demand for it will increase from week to week. Initially, in the first week, he sells 1000 units of the good. He has two possible models of how sales might increase each week. These are: (i) weekly sales rise by 100 units each week; and (ii) weekly sales increase by 2% each week. In each case, find a formula for the number of units sold after N weeks.

10 (a) The supply function for a commodity is believed to take the form

$$q^S(p) = \frac{a}{p^2} + \frac{b}{p} + cp,$$

for some constants a, b, c. When $p = 1$, the quantity supplied is 13; when $p = 2$, the quantity supplied is 18; and when $p = 3$, the quantity supplied is 19. Find a system of linear equations for a, b, c. By using a matrix method to solve this system, find a, b and c.

(b) The function $f(x, y)$ is given by

$$f(x, y) = \left(\frac{x - y}{x + y}\right)^n,$$

where $n > 0$. Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

11 A consumer has utility function

$$u(x_1, x_2) = 2\sqrt{x_1} + \sqrt{x_2}$$

for two goods, X_1 and X_2. (Here, x_1 and x_2 are, respectively, the amounts of X_1 and X_2 consumed.) Suppose that each unit of X_1 costs \$$p_1$ and each unit of X_2 costs \$$p_2$, and that the consumer has an amount M to spend on these two goods. By using the Lagrange multiplier method, determine the quantities x_1^* and x_2^* of X_1 and X_2 that maximise the consumer's utility function subject to the constraint on his budget. Determine also the corresponding value λ^* of the Lagrange multiplier. Suppose that $V = u(x_1^*, x_2^*)$ is the maximum achievable utility. What is the value of the marginal utility of income, $\frac{\partial V}{\partial M}$?

END OF PAPER