SIM
GLOBAL EDUCATION

SINGAPORE INSTITUTE OF MANAGEMENT

PRELIMINARY EXAM

PROGRAMME(S)	:	University of London Degree and Diploma Programmes (Lead College: London School of Economics & Political Science)
SUBJECT	:	**05A MATHEMATICS 1 (HALF-UNIT)**
DATE	:	Tuesday, 26 February 2008
DURATION	:	2 hours

INSTRUCTIONS :-

DO NOT TURN OVER THIS QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Candidates should answer **NINE** of the following **ELEVEN** questions:
SEVEN from Section A (60 marks in total) and
TWO from Section B (20 marks each).

Graph paper is provided. If used, it must be fastened securely inside the answer book.

Calculators may **not** be used for this paper.

Candidates are strongly advised to divide their time accordingly.

Total number of pages : 4 (including this page)

SECTION A

Answer all **SEVEN** questions from this section (60 marks in total)

1. Sketch the graphs of the functions f and g where

$$f(x) = -x^2 + 3x + 10 \qquad \text{and} \qquad g(x) = 2x + 4,$$

and determine their points of intersection.

2. Find and classify the stationary points of the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 9$.

3. **Use a matrix method** to find the numbers x, y and z that satisfy the equations
$x + y + z = 4$, $2x - y - z = 2$ and $-x + y + 2z = 3$.

4. Suppose that the constant $a \neq -2, 0, 2$. Show that the only critical point of the function

$$f(x, y) = x^2 + axy + y^2,$$

is $(0, 0)$. Determine, for each possible value of a, the nature of this critical point.

5. Determine the following integrals.

(a) $\displaystyle\int x^n \ln(x)\, \mathrm{d}x$, where $n \neq -1$.

(b) $\displaystyle\int \frac{\cos x}{1 - \sin^2 x}\, \mathrm{d}x$.

6. Find the minimum value of $\dfrac{1}{x} + \dfrac{1}{y}$ for positive x and y satisfying $x^2 + y^2 = 8$.

7. Given that $V(x, y) = x\mathrm{e}^{x+ay}$, find the partial derivatives $\dfrac{\partial V}{\partial x}$ and $\dfrac{\partial V}{\partial y}$.
Suppose that V satisfies

$$\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} = \left(3 + \frac{1}{x}\right) V,$$

find the value of the constant a.

SECTION B

Answer **TWO** questions from this section (20 marks each)

8. (a) For $0 \leq q \leq 5$, a firm has a marginal cost function given by

$$MC(q) = 3q^2 + 36q - 36,$$

and its fixed costs are 6. If its revenue function is

$$2q^3 + 6q^2 + 9q,$$

find the profit function of the firm and determine the value of q that gives the maximum profit.

(b) Find the positive number y which is such that

$$\int_1^y \left(1 + \frac{2}{x^2}\right) \, dx = 2.$$

9. (a) Find the critical points of the function

$$f(x, y) = 3x^3 + 9x^2 - 72x + 2y^3 - 12y^2 - 126y + 19,$$

and determine their natures.

(b) The numbers x, y, z and a are related by the equations

$$\begin{aligned} x + 2y - z &= 3 \\ -x - 3y + z &= -5 \\ 2x + y - z &= a \end{aligned}$$

Using a matrix method, find expressions for x, y, z in terms of a.

10. (a) A village initially contains 100 women. Each year, of the women who were in the village at the start of the year, 20% give birth to a daughter and 10% of the women die. Additionally, each year, 5 new women come to the village from the surrounding countryside. How many women will there be in the village after t years?

(b) A firm is the only producer of two goods, X and Y. The demand functions for X and Y are, respectively, given by

$$x = 8(p_Y - p_X) \qquad \text{and} \qquad y = 4(9 + 2p_X - 4p_Y),$$

where p_X and p_Y are the corresponding prices. If it costs $1 to produce one unit of X and $1.50 to produce one unit of Y, find an expression for the profit function of the firm in terms of x and y. Hence determine the quantities x and y that maximise the profit.

11. Given an amount of capital, k, and an amount of labour, l, a firm produces a quantity $q(k, l) = k^\alpha l^\alpha$ where α is a constant such that $0 < \alpha < 1/2$. Each unit of capital costs v and each unit of labour costs w.

(a) Show that the minimum amount the firm can spend on capital and labour, if it is to manufacture an amount Q, is given by

$$2\sqrt{vw}Q^{1/(2\alpha)}.$$

The product manufactured by the firm sells at a fixed price of p per unit and the raw materials required to produce each unit cost an amount r where $r < p$.

(b) Assuming that the firm is acting so as to minimise its capital and labour costs, use the result above to find an expression for the profit made by the firm if it sells an amount Q.

(c) Find the amount, Q, that will maximise the firm's profit. (Make sure that the assertion that the profit is maximised is justified.)