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UNIVERSITY OF LONDON

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BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Mathematics 1 (half unit)

Wednesday, 11 May 2005: 10.00am to 12.00noon

Candidates should answer NINE of the following ELEVEN questions: SEVEN from Section A (60 marks in total) and TWO from Section B (20 marks each). Candidates are strongly advised to divide their time accordingly.

Graph paper is provided. If used, it must be securely fastened inside the answer book.

Calculators may not be used for this paper.

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SECTION A

Answer all seven questions from this section (60 marks in total).

- 1. On the same diagram, sketch the curves with equations $y = 2x^2 x 3$ and $y = 1 + x 2x^2$, indicating where each curve crosses each of the axes. Determine the values of x for which the two curves intersect.
- 2. The function f is defined, for $x \geq 0$, by

$$f(x) = e^{\sqrt{x}} - 2\sqrt{x}.$$

Find the critical point of f and show that it is a local minimum. What is the minimum value of f(x)?

3. Determine the integral

$$\int \frac{1}{1+e^x} \, dx.$$

- 4. Use a matrix method to find the numbers x, y, z that satisfy -x + 2y + 3z = 3, 2x y + 2z = 5 and x + y + z = 4.
- 5. Find the critical points of the function

$$f(x,y) = x^4 + 2x^2y + 2y^2 + 2y$$

and determine, for each, whether it is a local maximum, a local minimum, or a saddle point.

- 6. Use the Lagrange multiplier method to determine the values of x and y that minimise x + 2y subject to the constraint $x^3y^2 = 27$.
- 7. A saver has a bank account that pays 5% annual interest, paid on the last day of each year. He initially invests an amount \$A\$ and at the end of each year, immediately after the interest has been credited to his account, he withdraws an amount W. Find an expression, in as simple a form as possible, for the amount of money in the account at the end of N years.

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SECTION B

Answer two questions from this section (20 marks each).

- 8.(a) A firm is a monopoly and it has fixed costs of 5 and average variable cost function $q^4 q 5q^2$. The demand equation for the good it produces is p + q = 20, where p is the selling price. Find an expression for the firm's profit, as a function of q, and determine the production level that maximises the firm's profit.
 - (b) Determine the following integrals:

$$\int \frac{x}{(x+1)^3} dx$$
$$\int \frac{\ln x}{x((\ln x)^2 + 1)} dx.$$

9.(a) The function f(x) takes the form

$$f(x) = ax^{2} + b + \frac{c}{(1+x)^{2}},$$

for some constants a, b, c. The following facts about f and its derivative f' hold:

$$f(1) = 5/4$$
, $f'(1) = 15/4$, $\int_0^1 f(x) dx = 1/6$.

Using this information, show that the following system of equations holds for a, b and c:

$$4a + 4b + c = 5$$

 $8a - c = 15$
 $2a + 6b + 3c = 1$

Solve this system of equations using a matrix method, to determine a, b and c.

(b) The function f(x, y) is given by

$$f(x,y) = xy \ln \left(\frac{x}{y}\right),\,$$

for x,y>0. Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

- 10.(a) A consumer has utility function $u(x,y) = (x+2)^2(y+1)^3$ for two goods, X and Y. (Here, x denotes the amount of X and y the amount of Y.) Each unit of x costs p dollars and each unit of Y costs q dollars, and the consumer has a budget for X and Y of M dollars. Use the Lagrange multiplier method to find the quantities of X and Y the consumer will consume in order to maximise his utility subject to the budget constraint. (Your answers will depend on p, q and M.)
 - (b) A firm is the only supplier of two goods, X and Y, and the demand equations for these goods are

$$x = 76 - p_X, \quad y = 50 - \frac{1}{2}p_Y,$$

where p_X and p_Y are the prices of X and Y, and where x, y denotes (respectively) the amounts of X and Y. The firm has a total joint cost function

$$TC = 2x^2 + 2xy + 3y^2 + 10.$$

Determine an expression, in terms of x and y, for the firm's profit. Determine also the values of production x and y that will give maximum profit to the firm.

11.(a) A firm produces a good using two raw materials, X and Y, and the corresponding costs per unit of these raw materials are c_X and c_Y , respectively. The amount of its good that a firm can produce using x units of X and y of Y is $\sqrt{x} + \sqrt{y}$. Determine the minimum combined cost C of raw materials X and Y which will enable the firm to produce a total amount q of its good. Determine the value λ^* of the Lagrange multiplier corresponding to the optimising values of x and y. Show that

$$\lambda^* = \frac{\partial C}{\partial q}.$$

(b) A geometric progression is such that the sum of the first five terms is 31 and the sixth term is 16 times the second term. Determine all possible values for the common ratio and first term of the series.