This paper is not to be removed from the Examination Halls

UNIVERSITY OF LONDON

279 005a ZA
990 005a ZA
996 D05a ZA

BSc degrees in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Mathematics 1 (half unit)

Tuesday, 11 May 2004 : 10.00am to 12.00noon

Candidates should answer **NINE** of the following **ELEVEN** questions: **SEVEN** from Section A (60 marks in total) and **TWO** from Section B (20 marks each). **Candidates are strongly advised to divide their time accordingly.**

Graph paper is provided. If used, it must be securely fastened inside the answer book.

Calculators may **not** be used for this paper.

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UL04/407

SECTION A

Answer all **SEVEN** questions from this section (60 marks in total)

1 The functions $f(x)$ and $g(x)$ are given by

$$f(x) = 4x^2 - 8x - 1, \quad g(x) = -4x^2 - 2x + 4.$$

Sketch the graphs of $y = f(x)$ and $y = g(x)$ for $x > 0$ on the same diagram, and determine the positive value of x at which these two graphs intersect.

2 Find the value of x that maximises the function

$$f(x) = (1 + 2x)e^{-x^2}.$$

3 Determine the integral

$$\int \frac{x}{x^2 - x - 2}\, dx.$$

4 Find the critical point of the function

$$f(x, y) = x^2 - 2xy + 2y^2 - 2y + 2$$

and show that this critical point is a local minimum.

5 Use the Lagrange multiplier method to find the values of x and y that maximise the function $f(x, y) = xy^{3/2}$ subject to the constraint $x + 2y = 100$.

6 Express the following system of equations in matrix form, and solve it using a matrix method.

$$\begin{aligned} 4x + y - 2z &= 4 \\ 2x + 3y - 2z &= 4 \\ 2x + 5y + 2z &= 8. \end{aligned}$$

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7 A geometric progression has second term equal to 2 and a sum to infinity of 9. Show that there are two possible values of the common ratio, and find these.

SECTION B

Answer **TWO** questions from this section (20 marks each)

8 (a) A firm has average variable cost

$$q^2 + 2q + \frac{\ln(q^2+1)}{q}$$

and fixed costs of 9. Find the total cost function and the marginal cost function.

(b) Determine the integrals $\int x^2\sqrt{x+3}\,dx$ and $\int \frac{\ln x}{x^2}\,dx$.

(c) The function f is given by

$$f(x,y) = x^{-y}$$

for $x > 0$. Find the partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

9 (a) A monopolist's average cost function is

$$\frac{40}{q} + 20 - 3q + \frac{q^2}{2},$$

where q is the quantity produced. The inverse demand function for the good is

$$p = 40 - \frac{5}{2}q.$$

Find an expression for the profit in terms of q and determine the value of q that maximises the profit.

(b) A firm has production function given by

$$q(k,l) = k^{1/4}l^{1/4},$$

where k and l denote, respectively, capital and labour. Each unit of capital costs \$1 and each unit of labour costs \$16. Suppose that, when producing any given amount, the firm minimises its total expenditure on capital and labour. Show that when the production level is q, this minimum total expenditure on capital and labour is $8q^2$.

10 (a) Three goods are sold in the same market. If their prices are p_1, p_2, p_3, then the demand quantities q_1^D, q_2^D, q_3^D and the supply quantities q_1^S, q_2^S, q_3^S are given by the following equations.

$$\begin{aligned} q_1^D &= 48 - 2p_1 + 2p_2 - 4p_3 \\ q_1^S &= 2p_1 - 2 \end{aligned}$$

$$\begin{aligned} q_2^D &= 10 + 2p_1 - p_2 + 4p_3 \\ q_2^S &= 2p_2 - 10 \end{aligned}$$

$$\begin{aligned} q_3^D &= 20 - p_1 + 2p_2 - 2p_3 \\ q_3^S &= 2p_3 - 15. \end{aligned}$$

The equilibrium prices are the non-negative numbers p_1^*, p_2^*, p_3^* with the property that when the prices are $p_1 = p_1^*, p_2 = p_2^*$ and $p_3 = p_3^*$, then the supply and demand quantities for each good are equal. **Using matrix methods**, find p_1^*, p_2^*, p_3^*.

(b) A firm has marginal cost function $1 + e^{0.5q} + q^2$ and fixed costs of 10. Find the total cost function.

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11 (a) A holiday company organises unique vacations in a particular resort during two periods of the year: the peak period and the off-peak period. The demand equation relating vacation price p_1 and the demand q_1 of vacations in the peak period is $p_1 + 25q_1 = 1050$. For the off-peak period, with price denoted p_2 and demand by q_2, we have $p_2 + 2q_2 = 500$. The total annual cost to the company of providing the vacations is given by $TC = 10 + 50q_1 + 20q_2$.

Find an expression for the annual profit in terms of q_1 and q_2.

Determine the values of q_1 and q_2 which maximise profit to the company.

(b) On the first day of 2004 in the Republic of Utopia there are 1000 on-line book retailers. During each subsequent year, the number of new such retailers grows by 30, but by the end of the year 2% of all the on-line book retailers that were in business at the start of the year will have closed down. Find an expression, in terms of N (and in as simple a form as possible) for the number of on-line book retailers N years after the first of January 2004. What happens to the number of such retailers in the long run?

END OF PAPER