This paper is not to be removed from the Examination Halls

UNIVERSITY OF LONDON

279 005b ZA
990 005b ZA
996 D05b ZA

BSc degrees and Diplomas for Graduates in Economics, Management, Finance and the Social Sciences, the Diploma in Economics and Access Route for Students in the External Programme

Mathematics 2 (half unit)

Wednesday, 11 May 2005 : 2.30pm to 4.30pm

Candidates should answer **EIGHT** of the following **TEN** questions: **SIX** from Section A (60 marks in total) and **TWO** from Section B (20 marks each).

Graph paper is provided. If used, it must be securely fastened inside the answer book.

Calculators may **not** be used for this paper.

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SECTION A

Answer all **six** questions from this section (60 marks in total)

1. The supply equation for a good is $q = 2p$ and the demand equation is $q = 2400 - p$, where q denotes quantity and p the price in dollars. Determine the equilibrium price and quantity. Suppose a percentage tax of 25% is imposed. Determine the new equilibrium price and quantity. Determine also what excise (per-unit) tax would result in the same equilibrium price.

2.(a) The function $f(x, y)$ takes the form

$$f(x,y) = \frac{x^\alpha y^{-1} + x^2 y^\beta}{\sqrt{x^4 + y^4}},$$

for some numbers α, β and γ. If f is homogeneous of degree D, what must the values of α, β and γ be (in terms of D)?

(b) Show that the function $f(x, y) = y \cos\left(\frac{x}{y}\right)$ is homogeneous, and verify explicitly that it satisfies Euler's equation.

3. The demand equation for a good is $p = \dfrac{210}{q^2 + 5q + 6}$ and the equilibrium price is 5. Determine the equilibrium quantity and the consumer surplus. If the elasticity of demand for the good is equal to 2 for every value of the price, determine the demand function.

4. Use a matrix method to find all solutions to the following system of equations

$$\begin{aligned} 2x - y + z &= 4 \\ -x + 3y + 2z &= 3 \\ x + 2y + 3z &= 7 \\ x - 2y - z &= -1. \end{aligned}$$

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5. A sequence x_t satisfies

$$x_{t+1} = ax_{t-1}$$

for all $t \geq 1$, where $a > 0$ is a fixed number. If $x_0 = 1$ and $x_1 = 3\sqrt{a}$, find a formula (in terms of t and a) for x_t.

6. The supply and demand functions for a good are, respectively,

$$q^S(p) = 2p, \quad q^D(p) = 8 - 2p.$$

Assuming that the initial price is $p(0) = 1$, and that the price adjusts over time according to the equation

$$\frac{dp}{dt} = \left(q^D(p) - q^S(p)\right)^2,$$

find a formula for $p(t)$. How does $p(t)$ behave as t tends to infinity?

SECTION B

Answer **two** questions from this section (20 marks each).

7.(a) Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$, where A is the matrix

$$\begin{pmatrix} 5 & 1 \\ 2 & 4 \end{pmatrix}.$$

Hence, or otherwise, find the functions $f(x)$ and $g(x)$ which are such that $f(0) = 2$, $g(0) = 1$ and

$$\begin{aligned} \frac{df}{dx} &= 5f(x) + g(x) \\ \frac{dg}{dx} &= 2f(x) + 4g(x). \end{aligned}$$

(b) Expand as a power series, in terms up to x^4, the function given (for $x < 1$) by $f(x) = \cos(\ln(1 - x))$.

8.(a) Sequences C_t, I_t, G_t and Y_t are related as follows:

$$\begin{aligned} C_t &= \frac{1}{8}Y_{t-1} \\ I_t &= 100 + \frac{1}{32}(Y_{t-1} - Y_{t-2}) \\ G_t &= \frac{3}{32}Y_{t-1} \\ Y_t &= C_t + I_t + G_t. \end{aligned}$$

Find a second-order difference equation for Y_t. Solve this equation to determine Y_t if $Y_0 = 129$ and $Y_1 = 128.25$.

(b) Consider the following system of equations.

$$\begin{aligned} x + y - 3z &= 4 \\ 2x - y + z &= 3 \\ x + 4y + az &= b. \end{aligned}$$

Use matrix methods to determine what values a and b must take if this system is consistent and has infinitely many solutions.

What must the value of a *not* be if the system has precisely one solution?

What can be said about a and b if the system has no solutions?

9.(a) Find the inverse of the matrix

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \end{pmatrix}.$$

(b) The function $f(x)$ satisfies

$$\frac{d^2f}{dx^2} - af = \frac{df}{dx} + a^2f + 1 + a^2x + ax$$

where $a > 1$ is some fixed number. Furthermore, $f(0) = 0$ and df/dx equals $-2a - 1$ when $x = 0$. Find the function $f(x)$.

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10.(a) Determine the minimum value of

$$\frac{1}{xy^2z^3}$$

subject to the constraint $px + qy + rz = c$, where $p, q, r, c > 0$ are fixed numbers.

(b) Suppose that the supply and demand equations for a particular market are, respectively,

$$p - 3q = 12, \quad p + q = 20.$$

Determine the supply function $q^S(p)$, the inverse supply function $p^S(q)$, the demand function $q^D(p)$ and the inverse demand function $p^D(q)$.

Suppose the suppliers operate according to the cobweb model, so that if p_t and q_t are (respectively) the price and quantity in year t, then $p_t = p^D(q_t)$ and $q_{t+1} = q^S(p_t)$. Suppose also that the initial price is $p_0 = 10$. Find an expression for p_t. How does p_t behave as t tends to infinity? How does q_t behave as t tends to infinity?

END OF PAPER