Task1 : Pass Grade – P3.1

(A) The electric potential produced by a uniform spherical volume charge of charge Q and radius r_0 is :

$$V = \begin{cases} \dfrac{Q}{4\pi\varepsilon_0 r_0^3}(3r_0^2 - r^2) & \text{for} \quad r < r_0 \\[2ex] \dfrac{Q}{4\pi\varepsilon_0 r} & \text{for} \quad r > r_0 \end{cases}$$

Where r is the distance from the center of the charge at which an electric field is created and ε_0 is the permittivity constant of vacuum.
Use this potential to determine the electric field due to a uniform spherical volume charge for the following two cases:

1. The field inside the charge distribution.
2. The field outside the charge distribution.

(B) The charge on the capacitor in an RLC circuit is given by :

$$q = Q_m e^{\frac{-t}{\tau}} \cos(w_d t + \phi)$$

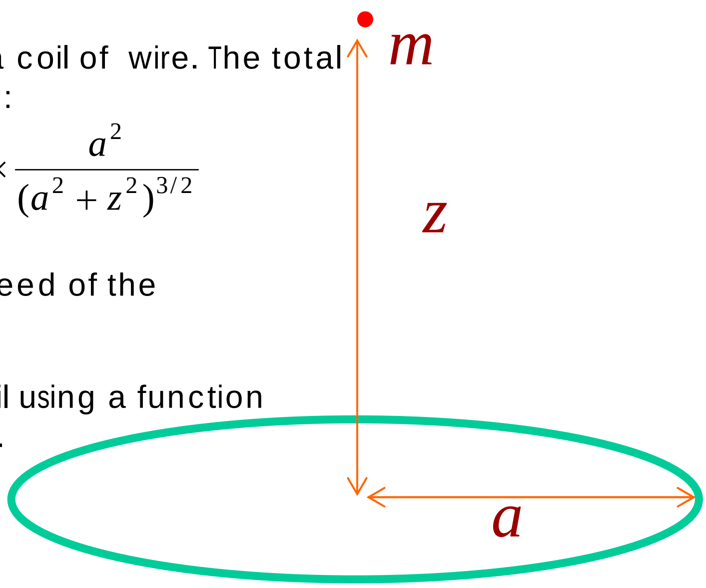
Where Q_m is the maximum charge of the capacitor , w_d is the *damped* angular frequency of the oscillation , ϕ is the phase constant , ($w_d t + \phi$) is the phase and τ is a constant (equals 2L/R).
Find the current i in the circuit.

(C) A magnet is dropped through a coil of wire. The total flux through the loop is given by :

$$\Phi = -\frac{\mu_0 \, mN}{2} \times \frac{a^2}{(a^2 + z^2)^{3/2}}$$

Where $z = vt$ and v is the speed of the falling magnet.

Find the induced emf in the coil using a function of a function differentiation rule.



Task2 : Pass Grade – P3.2

(A) The electric current charging a capacitor is related to time by the following Laws**:**

1. $I = t^4 + \arcsin(t^2) + e^t \ln t + 7$
2. $I = \cos(\ln t) + \dfrac{t+1}{2t+3}$
3. $I = \arcsin(4t) + \arccos t$
4. $I = a\tanh(t^2)$
5. $I = t^2 - \ln(\sqrt{2}\,\mathrm{t}) + \sinh(t^2 + 5)$

Determine $\dfrac{dI}{dt}$ and $\dfrac{d^2I}{dt^2}$ for the above five laws.

(B) The current in an RLC circuit is given by :

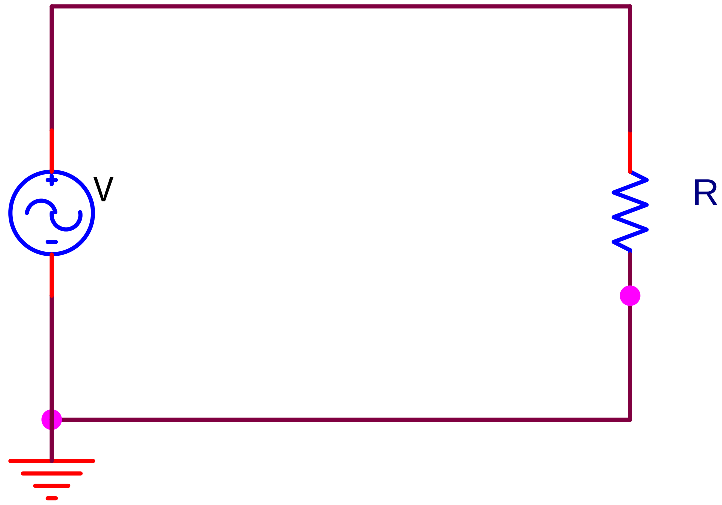
$I = a\sinh(4t)$ Find $\dfrac{d^4I}{dt^4}$

Task3 : Pass Grade – P3.3

(A) For AC circuit, voltage across a resistor can be written as

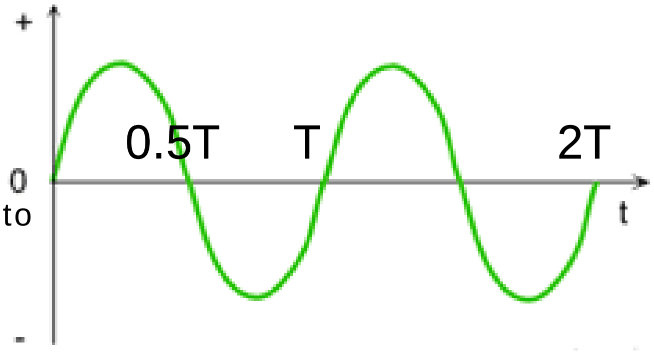
$$V(t) = V\sin(2\pi f t)$$

where f is frequency and V is the amplitude in volt.



To compute the average power delivered to a resistor from an AC source, we have to consider the time average of the AC source first:

$$\langle V^2(t)\rangle = \frac{\int_0^T V^2(t)\,dt}{T}$$



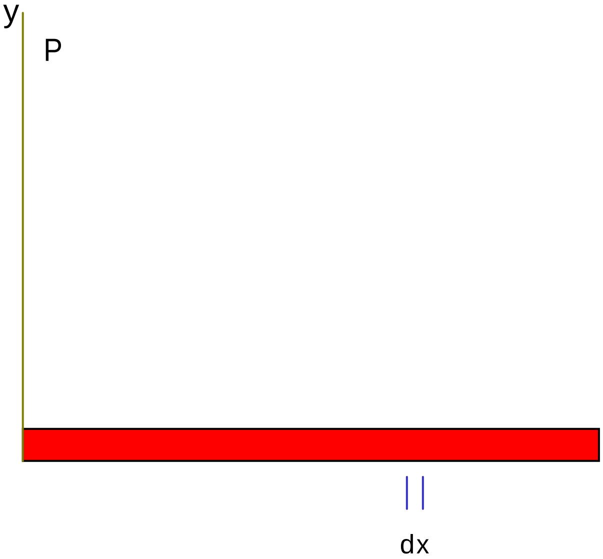
Knowing that the average power P delivered to the resistor is related to the time average as:

$$P = \frac{\langle V^2(t)\rangle}{R}$$

Determine P in terms of V and R

(B) The potential due to a finite line is given by

$$V = \frac{k_e Q}{l}\int_0^l \frac{dx}{\sqrt{x^2 + a^2}}$$



Determine an expression of V in terms of l and a using integration by parts.

(C) The electric current charging a capacitor is related to time by the following Law**:**

$$i(t)=\frac{5t^2-2t+13}{(t+3)(t-1)^2}$$

Determine V_C knowing that: $V_C=\frac{1}{C}\int i\,(t)\,\mathrm{dt}$

Task4 : Pass Grade – P3.4

(A) The power output of a solar electricity device P as a function of the voltage V is given by :

$$P=\left(Je^{-\frac{e}{KT}V}\right)V$$

Where J is the current density that flows through the load, K and T are constants.

Calculate the voltage that will produce maximum power.

(B) The voltage V_C across a capacitor when it discharges through a resistance is given by:

$$V_0-V_C=k\frac{dV_C}{dt}$$

Where V_0 is the voltage at time t = 0 and k is a time constant.

Find the equation that relates V_C with t.